Depreciating Licenses*

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Abstract

Many kinds of resource use rights are assigned using perpetual or long-term licenses, in order to give license owners incentives to invest in and maintain the common value of the resource. However, long-term licenses are potentially harmful for allocative efficiency. Short-term licenses can improve allocative efficiency, but reduce incentives for investment. We propose an alternative licensing scheme: *depreciating licenses*. A depreciating license owner annually announces a valuation at which she commits to sell the license to any interested buyer, and pays a license fee based on this valuation. Depreciating licenses have no term limit, so they induce high and time-stationary investment incentives, but the value announcement scheme increases license turnover and improves allocative efficiency. Depreciating licenses are simple to implement, as the sole tuning parameter, the depreciation rate, can be chosen by targeting the observed equilibrium probability of license turnover. In a calibrated model, depreciating licenses increase average welfare from asset use by over 4% of asset prices relative to perpetual ownership licenses.

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1 Introduction

The role of property rights in promoting the efficient use of scarce resources is perhaps the oldest idea in mainstream economic thought. The intuition that property rights solve the "tragedy of the commons," giving agents incentives to invest in maintaining the common value of assets, dates back to the Greek prehistory of the field.[1] The importance of property rights for investment incentives is the subject of a sizable modern literature (Grossman and Hart, 1986; Hart and Moore, 1990)

On the other hand, private property can adversely affect the efficient allocation of assets. Myerson and Satterthwaite (1981) show that fully efficient trade of privately owned assets under asymmetric information is impossible. This is fundamentally a failure of property rights: if assets are initially owned by the government, full allocative efficiency can be achieved by selling assets in second-price auctions. In the mechanism design literature, Cramton, Gibbons and Klemperer (1987) and Segal and Whinston (2011) show that partial property rights mechanisms can similarly allow more efficient bargaining than full private ownership. This suggests that private ownership has important costs for allocative efficiency.

In this paper, we discuss the design of a certain kind of property right: use licenses for a variety of natural resources. This includes physical resources such as oilfields, fisheries, and timber, as well as nonphysical resources such as radio spectrum, airspace rights, and internet domain names. For many of these resources, both allocative and investment efficiency are important concerns. On the allocative side, different parties have different values and costs for using different assets. In the case of radio spectrum, different technologies which may have very different economic values, such as TV broadcasting, bluetooth, and wireless broadband, compete for the same spectrum use rights. On the investment side, costly maintenance activities may be required to sustain the value of the resource for all parties. For example, radio spectrum use requires building base stations and antennae, and fishery use requires maintaining the ecosystem and preventing pollution.

In designing licenses to assign these resource use rights, the government faces a tradeoff between allocative and investment efficiency. Assigning use rights using long-lasting or perpetual licenses, approximating full private property rights, provides robust incentives for common-valued investment. However, perpetual licenses are potentially harmful for allocative efficiency. While the resource can be allocated efficiently in an initial license auction, the optimal user of the resource may shift over time; long-term licenses creates bargaining frictions for aftermarket trade which can hinder efficient reallocation of the resource.[2] Reallocation frictions can be reduced

[1] "What is common to the greatest number gets the least amount of care. Men pay most attention to what is their own; they care less for what is common; or at any rate they care for it only to the extent to which each is individually concerned." Aristotle, *The Politics*, Book XI, Chapter 3

[2] An example showing the difficulties involved in such reallocation is the recent FCC incentive auction, which reallocated long-lasting spectrum use licenses from TV broadcasters to wireless carriers (Leyton-Brown, Milgrom and Segal, 2017).

by shortening license term limits, running frequent auctions to reallocate resource use rights periodically to the highest-value party. However, a party who holds a short-term license for a resource has diminished incentives for common-valued investment. Intuitively, an effective license scheme should provide robust and long-term incentives for investment while promoting efficient aftermarket trade of use rights.

In this paper, we propose *depreciating licenses* as an alternative licensing scheme for these resources. The government initially sells a depreciating license for a given resource to bidders in an auction. The depreciating licenses has indefinite length, but is characterized by some annual depreciation rate τ. Each year, a share τ of the license reverts to the government; the license holder must announce a price at which she repurchases the share τ of the license from the government. We can alternatively think of the repurchase as a self-assessed license fee. License owners' price announcements are kept in a publically available register, and any interested party can purchase any license from its current owner at her current value announcement.

Depreciating licenses create high and stable incentives for common-valued investment, improving significantly upon term-limit licenses. Depreciating licenses also promote aftermarket trade and improve allocative efficiency by requiring license owners to announce valuations which serve as ask prices, and disciplining the ask prices through the self-assessed license fee. We show that the price-setting behavior of license holders is governed by a simple economic intuition – license owners set prices higher or lower than their true values depending on whether the annual probability of license sale is higher or lower than the depreciation rate. Thus, the equilibrium license turnover rate is an observable approximate sufficient statistic for setting optimal license depreciation rates.

We construct and numerically study a simple dynamic model to study the behavior of depreciating licenses in stationary trading equilibrium. In our model, a depreciating license for a single asset is repeatedly traded between many agents, who make common-valued investment and price announcement decisions in each period. We calibrate the model to loosely match moments of existing markets for durable assets, and find that depreciation rates set at roughly 2.5% are near optimal for a range of specifications and parameter values. This increases the net utility generated by the asset in the stationary trading equilibrium of the model by approximately 4% in our baseline specification, or roughly a fifth of a standard deviation in willingness-to-pay of different potential asset buyers.

The rest of the paper proceeds as follows. In Section 2, we illustrate the basic intuitions behind depreciating licenses in a simple two-stage model. In Section 3, we introduce the general dynamic model and characterize its equilibria. In Section 4, we calibrate the model to existing markets. In Section 5, we discuss various extensions, such as the effect of observability of investment, the effect of depreciating licenses on private-value investments, and relaxing our assumptions on the nature of buyers and sellers. In Section 6, we discuss our proposal's relationship to other work on mechanism design, asset taxation and intellectual property, as well as potential applications of our proposal. We conclude in Section 7. We present longer and less instructive calculations, proofs, and calibration details in an appendix following the main text.

2 Two-stage model

Relative to perpetual ownership licenses, depreciating licenses improve allocative efficiency at the cost of reducing investment incentives. This tradeoff is modulated by the choice of depreciation rate. We illustrate these intuitions in a simple two-stage model in which a depreciating license owner makes some costly common-valued investment in an asset, and then announces her valuation for the license, paying the self-assessed license fee and potentially selling the license to an arriving buyer.

2.1 Setup

There are two agents, S and B. There is a single asset, and S initially owns a depreciating license for the asset. Values of S and B for the asset are, respectively,

$$v_S = \eta + \gamma_S,$$

$$v_B = \eta + \gamma_B.$$

The term γ_S represents S's idiosyncratic value component for the asset; it is fixed and known to S at the beginning of the game. $\gamma_B \sim F(\cdot)$ is a random variable representing heterogeneity in B's value, which is not observed by S. η is a common-value component; S chooses $\eta \geqslant 0$, incurring a convex cost $c(\eta)$ to herself. Both agents are risk neutral.[3]

For a given η, let $1_S, 1_B$ be indicators, which respectively represent whether S and B hold the license at the end of the game, and let y be any net transfer B pays to S. Final payoffs for S and B respectively are

$$U_S = (\eta + \gamma_S) 1_S - c(\eta) + y,$$

$$U_B = (\eta + \gamma_B) 1_B - y.$$

Prior to the beginning of the game, the government decides on a depreciation rate τ. Then, S and B play a two-period game. In period 1, S chooses η. In period 2, S announces a price p for the license, pays the self-assessed license fee $p\tau$ to the government, and then B can decide whether to buy the license by paying p to S.

We solve the game by backwards induction. First, fixing η and τ, we analyze the behavior of S in the period 2 price offer game.

[3] See Tideman (1969) for a partial analysis of the allocative problem that allows for risk aversion.

2.2 Allocative efficiency

For any price p, B's optimal strategy is to buy the license if her value is greater than p, that is, if $\eta + \gamma_B > p$. Let $m \equiv p - \eta$ be the markup S chooses to set over the common value η. The probability of sale under markup m is then $1 - F(m)$. Fixing common value η, and depreciation rate τ, S's optimal price offer solves:

$$\max_m (1 - F(m))(\eta + m) + F(m)(\eta + \gamma_S) - \tau(\eta + m) - c(\eta).$$

We can change variables to work in terms of sale probabilities. Define $q \equiv (1 - F(m))$, and $M(q) \equiv F^{-1}(1 - q)$. S then solves:

$$\max_q (\eta + M(q))\, q + (\eta + \gamma_S)(1 - q) - \tau(\eta + M(q)) - c(\eta).$$

Note that the socially efficient outcome corresponds to setting $M(q) = \gamma_S$, or equivalently $q = 1 - F(\gamma_S)$. We can rearrange S's optimization problem to:

$$\max_q (M(q) - \gamma_S)(q - \tau) + (\eta + \gamma_S)(1 - \tau) - c(\eta).$$

Only the *variable profit* term $(M(q) - \gamma_S)(q - \tau)$ depends on the sale probability q. Thus, S's optimal choice of sale probability if her value is γ_S and the depreciation rate is τ can be written as:

$$q^*(\gamma_S, \tau) \equiv \arg\max_q (M(q) - \gamma_S)(q - \tau).$$

We can think of the objective function as the net trade profits of an agent who sells share q of the asset to buyers, and buy share τ of the asset from the government, both at price $M(q)$. In the following Theorem, we show that the relationship between τ and q summarizes license owners' incentives to markup or markdown prices.

Theorem 1. *(Net trade property)*

- *If* $\tau = 1 - F(\gamma_S)$*, then* $q^*(\gamma_S, \tau) = \tau$ *and* $M(q^*(\gamma_S, \tau)) = \gamma_S$.
- *If* $\tau < 1 - F(\gamma_S)$*, then* $q^*(\gamma_S, \tau) \geqslant \tau$ *and* $M(q^*(\gamma_S, \tau)) \geqslant \gamma_S$.
- *If* $\tau > 1 - F(\gamma_S)$*, then* $q^*(\gamma_S, \tau) \leqslant \tau$ *and* $M(q^*(\gamma_S, \tau)) \leqslant \gamma_S$.

Proof. First, suppose $\tau = 1 - F(\gamma_S)$.

- If S chooses sale probability $q = \tau$, she makes no net trades, and receives 0 variable profits. Moreover, the markup is $M(q) = F^{-1}(1 - \tau) = \gamma_S$, so that also $M(q) - \gamma_S = 0$.

- If S chooses a higher sale probability, so that $q - \tau > 0$, we have $M(q) \leqslant \gamma_S$, so variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$. In words, S becomes a net seller at a price lower than her value.

- Symmetrically, if S chooses a lower sale probability $q - \tau < 0$, she becomes a net buyer at a price higher than her value, and once again variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$.

Now suppose that $\tau < 1 - F(\gamma_S)$.

- By the first part of the Theorem, license owners with higher values $\gamma_S' = F^{-1}(1-\tau)$ have $\tau = 1 - F(\gamma_S')$, hence choose $q^*(\gamma_S', \tau) = 1 - F(\gamma_S^*)$. By construction, $\gamma_S \leqslant \gamma_S'$; since the variable profit function is supermodular in q and $-\gamma_S$, $q^*(\gamma_S, \tau) \geqslant q^*(\gamma_S', \tau) = \tau$.

- By the first part of the Theorem, if we set a lower rate $\tau' = 1 - F(\gamma_S)$, we have $q^*(\gamma_S, \tau') = 1 - F(\gamma_S)$ and $M(q^*(\gamma_S, \tau')) = \gamma_S$. By construction, $\tau \leqslant \tau'$. Since $M(q)$ is a decreasing function, the variable profit function is supermodular in q and τ, hence $q^*(\gamma_S, \tau) \leqslant q^*(\gamma_S, \tau') = 1 - F(\gamma_S)$. This implies that $M(q^*(\gamma_S, \tau)) \geqslant M(q^*(\gamma_S, \tau')) = \gamma_S$.

An analogous argument shows that $\tau > 1 - F(\gamma_S)$ implies that $q^*(\gamma_S, \tau) \leqslant \tau$ and $M(q^*(\gamma_S, \tau)) \leqslant \gamma_S$. □

Theorem 1 shows that the net effect of depreciating licenses on sellers' price-setting incentives is linked to an observable quantity: $\tau - q$, the difference between the depreciation rate and the probability of sale that it induces. If τ are lower than equilibrium sale probabilities, license owners can be thought of as selling a larger share of the asset than they are buying from the government, hence have net incentives to set prices higher than their values. Likewise, if τ is higher than sale probabilities, license owners are buying more from the government than they are selling, thus set prices below their values. When τ is equal to the probability of sale, asset owners are neither net buyers nor net sellers of their assets; thus they set prices equal to their values, leading to full allocative efficiency.

Theorem 2 of Section 3 shows that the net trade property generalizes to our dynamic model. In a setting with many license owners with heterogeneous values, no single depreciation rate can give all owners incentives to truthfully reveal their values; however, we show in our calibration of Section 4 that depreciation rates equal to average sale probabilities across sellers are close to allocatively optimal. Thus, the license turnover rate serves as an observable approximate sufficient statistic for setting depreciation rates.

We proceed to quantify the comparative statics of allocative welfare with respect to the depreciation rate. Assuming $F(\cdot)$ is twice continuously differentiable, S's first-order condition is

$$M'(q)(q - \tau) + (M(q) - \gamma_S) = 0,$$

so that by the Implicit Function Theorem,

$$\frac{\partial q^\star}{\partial \tau} = \frac{M'\left(q^\star\left(\gamma_S,\tau\right)\right)}{2M'\left(q^\star\left(\gamma_S,\tau\right)\right)+M''\left(q^\star\left(\gamma_S,\tau\right)\right)\left(q^\star\left(\gamma_S,\tau\right)-\tau\right)} = \frac{1}{2+\frac{M''\left(q^\star-\tau\right)}{M'}} = \frac{1}{2-\frac{M''\left(M-\gamma_S\right)}{\left(M'\right)^2}},$$

where the last equality invokes the first-order condition and drops arguments. Cournot (1838) showed that this quantity equals the *pass-through* rate $\rho\left(q^\star\left(\gamma_S,\tau\right)\right)$ of a specific commodity tax into price; see Weyl and Fabinger (2013) for a detailed discussion and intuition. ρ is closely related to the curvature of the value distribution; it is large for convex demand and small for concave demand. It is strictly positive for any smooth value distribution and is finite as long as S is at a strict interior optimum.[4]

The marginal gain to social welfare from a unit increase in the probability of sale is equal to the gap between γ_B and γ_S, which is by construction $\left(M\left(q^\star\left(\gamma_S,\tau\right)\right)-\gamma_S\right)$. Thus, the marginal allocative gain from raising τ is $\left(M\left(q^\star\left(\gamma_S,\tau\right)-\gamma_S\right)\right)\rho\left(q^\star\left(\gamma_S,\tau\right)\right)$, or $\left(M-\gamma_S\right)\rho$ for short. Since $\left(M-\gamma_S\right)\rho$ is 0 at $q = 1-F\left(\gamma_S\right)$, the first-order social welfare gain from raising τ approaches 0 as we approach the allocatively optimal turnover rate of $1-F\left(\gamma_S\right)$. On the other hand, when $\tau = 0$, we have $\left(M-\gamma_S\right)\rho > 0$, hence increasing τ from 0 creates a first-order welfare gain.

2.3 Investment efficiency

The variable profit term $\left(M\left(q\right)-\gamma_S\right)\left(q-\tau\right)$ in S's objective function is independent of η. Hence, S chooses investment to maximize the *sunk profit* term $\left(1-\tau\right)\eta - c\left(\eta\right)$. The first-order condition for this optimization problem is:

$$c'\left(\eta\right) = 1-\tau.$$

We can define the investment supply function $\Gamma\left(\cdot\right)$ as:

$$\Gamma\left(s\right) \equiv c'^{-1}\left(s\right).$$

The value of a unit of investment η is always 1, so the socially optimal level of investment is $\Gamma\left(1\right)$, whereas investment is only $\Gamma\left(1-\tau\right)$ when the depreciation rate is τ. The social value of investment is always 1, whereas S only invests up to the point where $c' = 1-\tau$. Thus, the marginal distortion from under-investment is τ. The marginal increase in investment from a rise in τ is $\Gamma' = \frac{1}{c''}$ by the inverse function theorem. Thus, the marginal social welfare loss from raising τ is $\Gamma'\tau$, or $\frac{\tau}{1-\tau}\Gamma\epsilon_\Gamma$, where ϵ_Γ is the elasticity of investment supply. Note that as $\tau \to 0$, this investment distortion goes to 0, so that there is no first-order loss in investment welfare when $\tau = 0$. Since there is a first-order gain in allocative welfare from raising the depreciation rate when $\tau = 0$, the optimal choice of τ is strictly greater than 0.

[4]Myerson (1981)'s regularity condition is sufficient but not necessary for this second-order condition, as we show in Appendix A.1.

2.4 Allocation-investment tradeoff

Figure 2.4 graphically illustrates the tradeoff between allocative and investment welfare. Allocative welfare increases monotonically in τ on the interval $\tau \in [0, 1 - F(\gamma_S)]$. The marginal gain in allocative welfare from raising the depreciation rate is $(M(q^\star) - \gamma_S)\,\rho(q^\star)$; thus, the marginal allocative gain is 0 when $\tau = 1 - F(\gamma_S)$ and $M(q^\star) = \gamma_S$. Similarly, the marginal investment loss from taxation is $\Gamma'\tau$, which is 0 at $\tau = 0$. These properties hold independently of the cost function and demand distribution; intuitively, this reflects the fact that both the marginal trades when $\tau = 1 - F(\gamma_S)$, and the marginal units of investment when $\tau = 0$, have no social value. Thus, regardless of the underlying cost and demand functions, the efficient depreciation rate τ_{eff} lies strictly in the interior of the interval $[0, (1 - F(\gamma_S))]$.

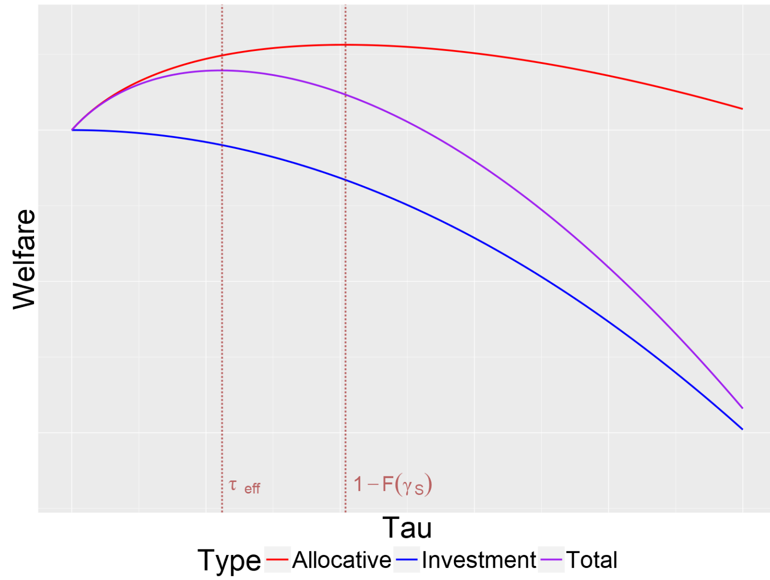
In Figure 2.4, allocative welfare is concave and investment losses are convex in τ, so total social welfare is a concave function of τ. While this is not true for all cost functions and demand distributions, it tends to hold for well-behaved values of the primitives. Since the markup $M(q^\star)$ is decreasing in τ, allocative marginal gains $(M(q^\star) - \gamma_S)\,\rho(q^\star)$ tend to be decreasing in τ, and since the marginal investment loss $\Gamma'\tau$ contains a τ term, marginal investment losses tend to be increasing in τ. Intuitively, as we raise τ from 0, the first trades that go through are the highest value trades, and the first investment losses are those which are both privately and socially marginal. As we raise τ, the allocative wedge $M(q^\star) - \gamma_S$ decreases, so new trades are less valuable, and the investment wedge τ increases, so the new investment losses are more costly to society. Thus, for relatively smooth demand forms and cost functions, the social optimization problem of maximizing the allocative gain less the investment loss will be concave.

In Appendix A.1, we formally derive sufficient conditions on the cost function $c(\cdot)$ and the inverse demand function $M(\cdot)$ for concavity of the social optimization problem. Under concavity, the following first-order condition, which resembles an optimal tax formula, uniquely characterizes the welfare-maximizing depreciation rate:

$$\frac{\tau_{eff}}{1 - \tau_{eff}} = \frac{(M(q^\star(\gamma_S, \tau_{eff})) - \gamma_S)\,\rho(q^\star(\gamma_S, \tau_{eff}))}{\Gamma(1 - \tau_{eff})\,\epsilon_\Gamma(1 - \tau_{eff})}. \tag{1}$$

The left-hand side is a monotone-increasing transformation of τ that appears frequently in elasticity formulas in the optimal tax literature; see, for example, Werning (2007). The right-hand side is the ratio of two terms: the allocative benefit of higher taxes and the investment distortion of higher taxes. The allocative benefit equals the product of the mark-up and the pass-through rate, whereas the investment distortion equals the product of the equilibrium investment size and its elasticity with respect to $1 - \tau$.

Figure 1: Allocative, Investment, and Total Welfare vs τ



Notes. Qualitative behavior of allocative (red), investment (blue) and total (purple) welfare vs. depreciation rate.

3 Dynamic model

In this section, we construct a simple dynamic extension of the two-stage model. While the economic intuitions are essentially identical to those of the preceding section, the dynamic model has well-defined quantities such as turnover rates, asset prices, and stationary value distributions, which we use to calibrate to data in Section 4.

3.1 Agents and utilities

Time is discrete, $t = 0, 1, 2 \ldots \infty$. All agents discount utility at rate δ. There is a single asset, which an agent S_0 owns at time $t = 0$. In each period, a single buyer B_t arrives to the market and bargains with the period-t owner S_t to purchase the license, through a procedure we detail in Subsection 3.2 below. Hence, the set of agents is $\mathscr{A} = \{S_0, B_0, B_1, B_2 \ldots\}$. We will use S_t as an alias for the period-t owner, who may be a buyer $B_{t'}$ from some period $t' < t$. We will use A to denote a generic agent in $\mathscr{A}$.

In period t, agent A has period-t use value γ_t^A for the asset. The values of entering buyers $\gamma_t^{B_t}$ are drawn i.i.d. from some distribution F. Values evolve according to a Markov process: for any agent A with period-t use value γ_t^A, her use value in the next period γ_{t+1}^A is drawn from the transition probability distribution $G(\gamma_{t+1} \mid \gamma_t)$.

Assumption 1. $F(M) = 1$ *for some finite* M.

Assumption 2. $\gamma_t > \gamma_t'$ *implies* $G(\gamma_{t+1} \mid \gamma_t) >_{FOSD} G(\gamma_{t+1} \mid \gamma_t')$.

Assumption 3. $G(\gamma' \mid \gamma)$ *is continuous and differentiable in* γ *for any* γ'.

In any period, there is a single user of the asset. Let 1_t^A denote agent A being the user of the asset in period A. Agent A's utility for ownership path 1_t^A and utility path γ_t^A, is:

$$\sum_{t=0}^{\infty} \delta^t \left[1_t^A \gamma_t^A + y_t^A\right].$$

Where, y_t^A is any net monetary payment made to agent A in period t.

3.2 Game

The license has some fixed depreciation rate τ. We define the following *dynamic depreciating license game*. At $t = 0$, agent S_0 owns the license, and observes her own use value $\gamma_t^{S_0}$ for the asset. In each period t:

1. **Buyer arrival:** Buyer B_t arrives to the market; his use value $\gamma_t^{B_t}$ is drawn from $F(\cdot)$, and is observed by himself but not the period-t seller S_t.

2. **Seller price offer:** The license owner S_t makes a take-it-or-leave-it price offer p_t to buyer B_t, and immediately pays license fee τp_t to the government.

3. **Buyer purchase decision:**

   - If B_t chooses to buy the license, she pays p_t to S_t. B_t becomes the period-t asset user, $1_t^{B_t} = 1$, and enjoys period-t use value $\gamma_t^{B_t}$ from the asset. B_t becomes the license owner in period $t+1$, that is, $S_{t+1} \equiv B_t$. Seller S_t receives payment p_t from B_t, and seller S_t leaves the market forever, with continuation utility normalized to 0.
   - If B_t chooses not to purchase the license, S_t becomes the period-t asset user, $1_t^{S_t} = 1$, and she enjoys period-t use value $\gamma_t^{S_t}$ from the asset. S_t becomes the license owner in period $t+1$, that is, $S_{t+1} \equiv S_t$. Buyer B_t leaves the market forever, with continuation utility normalized to 0.

4. **Value updating**: $\gamma_{t+1}^{S_{t+1}}$, the period $t+1$ value for owner S_{t+1}, is drawn from the distribution $G\left(\gamma_{t+1} \mid \gamma_t^{S_{t+1}}\right)$ according to her period-t value $\gamma_t^{S_{t+1}}$.

3.3 Equilibrium

Equilibrium in the dynamic depreciating license game requires that, in all histories, all sellers make optimal price offers, and all buyers make optimal purchase decisions. Since τ, F, G are constant over time, the problem is Markovian: the optimal strategies of buyers and sellers may depend on their types $\gamma_t^{S_t}, \gamma_t^{B_t}$ respectively, but not on the period t. Hence we can characterize equilibria of the game by a stationary value function $V(\gamma)$ which describes the value of being a type γ seller in any given period.

In any period t, we can think of S_t as choosing a probability of sale q_t, where buyers in period t make purchase decisions according to the inverse demand function $p(q_t)$. If the continuation value in period $t+1$ for seller type γ_{t+1} is $V(\gamma_{t+1})$, the optimization problem that S_t faces is:

$$\max_{q_t} q_t p(q_t) + (1-q_t)\left[\gamma_t + \delta \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma_{t+1}) \mid \gamma_t]\right] - \tau p(q).$$

Simplifying and omitting t subscripts, optimality for sellers requires $V(\gamma)$ to satisfy the following Bellman equation:

$$V(\gamma) = \max_{q} (q-\tau) p(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma') \mid \gamma\right]\right]. \tag{2}$$

Buyer optimality pins down the relationship between $p(\cdot)$ and $V(\cdot)$. If buyer B_t with value γ_t purchases the license, he receives value γ_t in period t, and then becomes the seller in period $t+1$, receiving utility $\delta V\left(\gamma_{t+1}^{B_t}\right)$. Hence the period-$t$ willingness-to-pay of buyer type γ_t is:

$$WTP(\gamma_t) = \gamma_t + \delta \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma_{t+1}) \mid \gamma_t].$$

Thus, in equilibrium, optimality for the buyer implies that the inverse demand function $p(\cdot)$ satisfies:

$$p(q) = \left\{p : \; P_{\gamma\sim F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma') \mid \gamma\right] > p\right] = q\right\}. \tag{3}$$

Fixing τ, a value function $V(\cdot)$ which satisfies Equations 2 and 3 defines an equilibrium of the dynamic depreciating license game.

Under Assumptions 1–3, we can prove that the net trade property from the two-stage depreciating license game applies to the dynamic game: net sellers set prices above their continuation values, and net buyers set prices below their continuation values.

Theorem 2. *(Dynamic net trade property) Under Assumptions 1–3, in any τ-equilibrium of the dynamic depreciating license game,*

- *For type γ with $\tau = 1 - F(\gamma)$, we have $q^*(\gamma) = \tau$ and $p(q^*(\gamma)) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$.*
- *For types γ with $\tau < 1 - F(\gamma)$, we have $q^*(\gamma) \geqslant \tau$ and $p(q^*(\gamma)) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$.*
- *For types γ with $\tau > 1 - F(\gamma)$, we have $q^*(\gamma) \leqslant \tau$ and $p(q^*(\gamma)) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$.*

Proof. See Appendix A.2. □

As we discuss in Appendix A.2, the problem is smooth enough that equilibria are guaranteed to exist. However, we were only able to prove uniqueness of equilibrium under an additional assumption:

Assumption 4. $\gamma_{t+1} \leqslant \gamma_t$ *with probability 1.*

The proof of equilibrium uniqueness given Assumption 4 is presented in the online appendix. This assumption is strong, but is satisfied in all specifications we use for our calibration. While we are unable to prove uniqueness without this assumption, in versions of the calibration in which $\gamma_{t+1} > \gamma_t$ with relatively small probability but decreases on average, we can numerically solve for equilibria and the equilibria appear to be unique. Thus, our calibration results do not appear to be very sensitive to Assumption 4.

3.4 Investment

Suppose that at the beginning of each period t, the current license owner S_t can make common-valued investment η_t in the asset at cost $c(\eta_t)$. As before, we assume that all investments are fully observable to all agents. We will allow investments to have long-term effects on common values: suppose that, in period $t' > t$, investment η_t increases the common value of the asset for all agents by some $H_{t'-t}(\eta_t)$. In Appendix A.3, we show that common-valued investment affects the equilibrium of the trading game by shifting all offered prices by some constant.

The social value of investment is the discounted sum:

$$\sum_{t=0}^{\infty} \delta^t H_t(\eta).$$

and, the social FOC sets:

$$c'(\eta) = \sum_{t=0}^{\infty} \delta^t H_t'(\eta).$$

The following proposition states that depreciating licenses distorts longer term investments more than shorter-term investments. Intuitively, if license owners make investments that pay off t periods in the future, they have to pay license fees for $t+1$ periods on their investments, generating an investment wedge of $(1-\tau)^{t+1}$ relative to the social optimum.

Proposition 1. *In any τ-equilibrium of the dynamic depreciating license game, all agents choose a constant level of investment η such that:*

$$c'(\eta) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H_t'(\eta).$$

Proof. See Appendix A.3. □

4 Calibration

In this section, we computationally solve our dynamic model under parameters chosen to match moments of various markets for durable assets.

4.1 Methodology

The dynamic depreciating license game requires us to specify a number of unknowns: the discount rate δ, the distribution of entering buyer values $F(\gamma)$, the transition probability distribution $G(\gamma' \mid \gamma)$, the investment cost function $c(\eta)$, and the investment benefit functions $H_t(\eta)$.

We use the standard choice of annual discount rate $\delta = 0.95$. We assume that the distribution of entering buyer values $F(\cdot)$ is log-normal, with log mean normalized to 0. The log standard deviation σ serves the role of a spread parameter, controlling the amount of idiosyncratic dispersion in values. While this F does not satisfy boundedness, as required by Assumption 1, we will approximate F using a bounded grid distribution.

We will use a variety of stochastic decay processes for the transition distribution $G(\gamma' \mid \gamma)$. In our baseline calibration, we will assume that if an agent has value γ_t in period t, her period $t+1$ value is $\chi\gamma$, where χ has a beta distribution with mean β. Thus, values decay by a factor β in expectation in each period. We will also show results from a variety of specifications using bimodal mixtures of such beta distributions. Finally, we use an extreme specification in which values either stay constant or jump to 0 with some probability. The choices of stochastic decay distributions are designed to modify the dispersion in the stationary distribution of seller values; our results show that this dispersion appears to be a key driver of the gains from depreciating licenses.

Given these choices for F and $G(\gamma' \mid \gamma)$, the parameters to be determined are the log standard deviation σ of F, and various shape parameters for the decay processes $G(\gamma' \mid \gamma)$. For each transition distribution, we fix all but one parameter affecting the mean of $G(\gamma' \mid \gamma)$ in advance; these choices for different specifications of $G(\gamma' \mid \gamma)$ are described in Appendix B.3. We then determine σ and the parameter moving the mean of $G(\gamma' \mid \gamma)$ by matching two empirical moments. First, we match the dispersion of buyer valuations to the dispersion of bids in various static auction settings. A large empirical literature studies static auctions for various use rights for government resources; these papers tend to find fairly high dispersion in the willingness-to-pay of different buyers for identical assets. The ratio of the standard deviation of idiosyncratic buyer values to its mean is found to be roughly 0.5 for timber auctions (Athey, Levin and Seira, 2011), 0.18 for highway procurement contracts (Krasnokutskaya and Seim, 2011), and roughly 0.2 for oil drilling rights (Li, Perrigne and Vuong, 2000). We will thus require, conservatively for our estimates of welfare gains, that in our model that the standard deviation of the allocative component of equilibrium willingness-to-pay of entering buyers is 0.20 times its mean.

Second, we target an annual turnover rate of 5% in equilibrium when $\tau = 0$. As turnover rates for resource rights are likely to be fairly heterogeneous, this is chosen somewhat arbitrarily. However, a 5% turnover rate is likely to be fairly low for many of the assets we consider, hence this choice should give a conservative estimate the extent of allocative distortions, and thus the welfare gains from depreciating licenses.

Increasing the rate of value decay lowering the mean of $G(\gamma' \mid \gamma)$ should increase the efficient probability of sale, the equilibrium probability of sale, as well as the optimal license depreciation rate. Increasing the lognormal standard deviation σ should increase the dispersion of values, the dispersion of prices, and the total achievable allocative welfare gains. Thus, intuitively, the $saleprob$ moment should be matched mostly by the parameter controlling the posterior mean of $G(\gamma' \mid \gamma)$, while the $sdmean$ moment should be matched mostly by σ. We confirm these intuitions for the baseline specification in Figure 4, which we discuss further in Subsection 4.2.

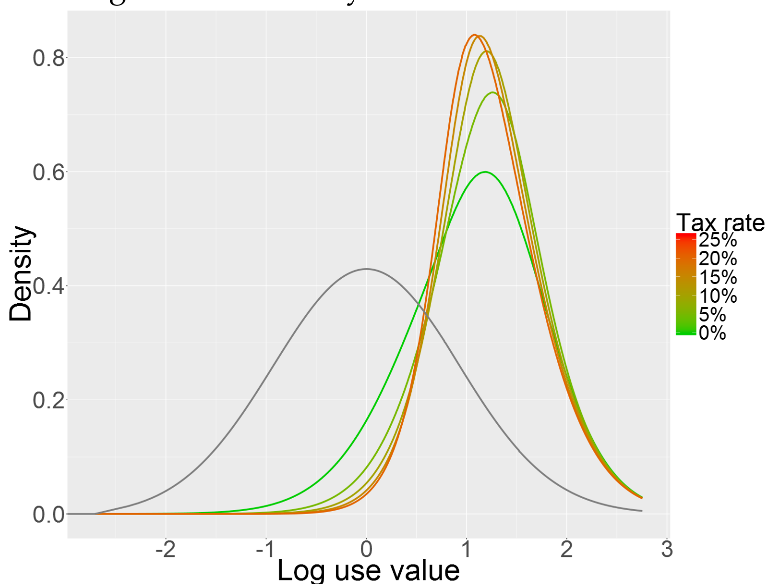
We will assume that investment has geometrically depreciating value over time: $H_t(\eta) = \theta^t \eta,\ \theta < 1$. For our calibrations, we will set $\theta = 0.85$, which is similar to depreciation rates from in the literature on capital depreciation (Nadiri and Prucha, 1996). Optimal depreciation rates and welfare gains turn out not to be very sensitive to the total value of investment welfare relative to allocative welfare. We will show this by reporting results for our baseline specification assuming that investment value constitutes a fraction 0%, 10%, 40% or 70% of total average asset value. In Appendix B.1, we derive analytical expressions for the equilibrium investment level and investment welfare for any value of τ. In Appendix B.2, we describe further details of the numerical procedure we use to analyze the game and solve for equilibria.

4.2 Results

In Figure 2, we plot the equilibrium stationary distribution of use values for different values of τ in the baseline specification. As we increase τ from 0 to 15%, probability mass moves from relatively low values towards higher values, as a result of lower markups and increased frequency of sales to high-value entering buyers. However, starting at around 5%, increasing τ also moves mass from the highest values towards somewhat lower values, though this effect does not become pronounced until τ reaches 15%. Intuitively, this is because the highest value license owners set prices below their values, causing the license to occasionally be purchased by buyers with values lower than that of the owner.

In Figure 3, we show the behavior of various quantities in stationary equilibrium as functions of τ, assuming that investment is 40% of total asset value. The topmost panel shows allocative, investment and total welfare, in units of percentages of the average license transaction price when the depreciation rate is 0. Allocative welfare is maximized at a $\tau = 8.7\%$, which optimizes the trade-off between moving mass away from low value quantiles and away from the highest quantiles. The horizontal line labeled $eff\_alloc\_welfare$ represents the max possible allocative welfare, calculated by solving for the steady-state distribution of use values assuming that the

Figure 2: Stationary Use Value Distribution



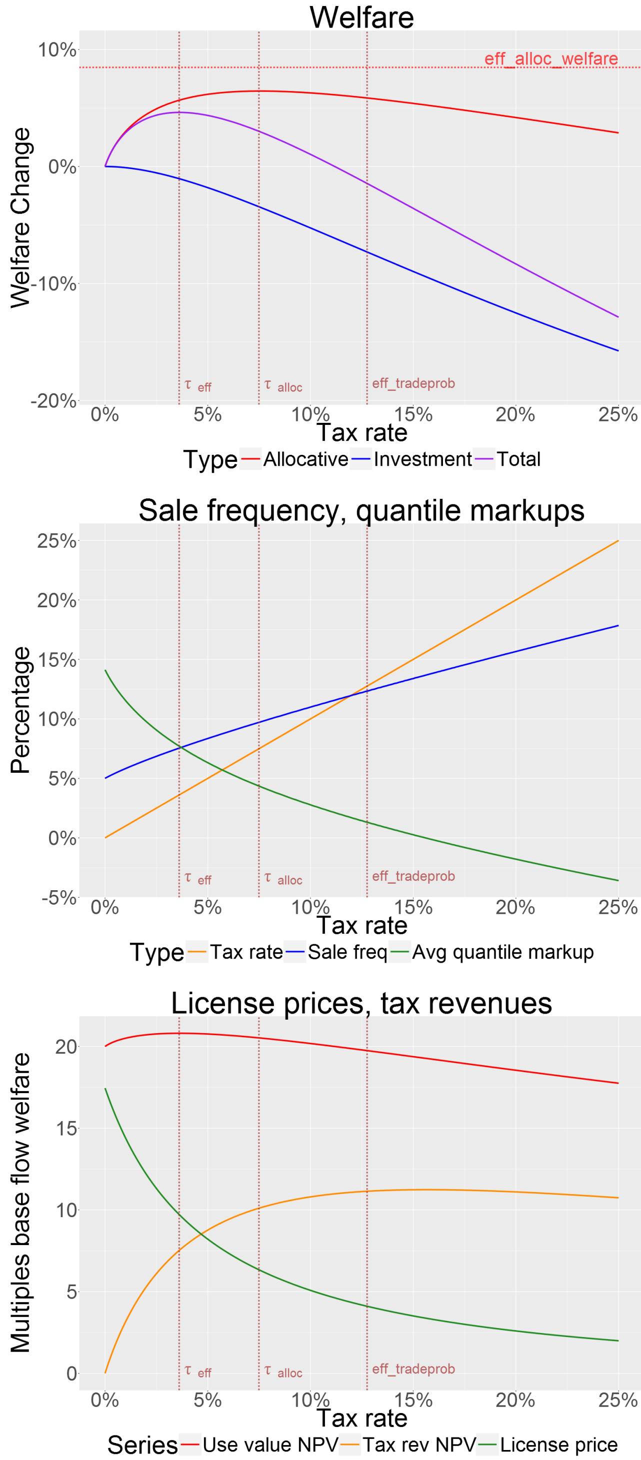
Notes. Stationary distributions of log use values in trading equilibrium, for different values of the depreciation rate τ. The gray line shows the distribution of entering buyers, which is lognormal with log mean 0.

asset is always transferred to the agent with higher value in any period. The allocatively optimal depreciation rate τ_{alloc} achieves over 80% of the total possible allocative welfare gains. If we take into account investment welfare, the optimal depreciation rate is 4.0%, and this increases total welfare by 5.4% of the baseline average asset price. Investment losses are not globally convex, likely due to the complex interactions of depreciating licenses with persistent investment. However, allocative welfare is still concave, and thus total social welfare is also concave in τ for depreciation rates below and near the efficient depreciation rate.

The second panel shows the equilibrium sale frequency and the average quantile markup set by sellers, as well as a line of slope 1 representing the depreciation rate τ itself. When the depreciation rate is set equal to the efficient probability of trade, the equilibrium average trade probability is also equal to the efficient probability of trade, and the average quantile markup is near 0. However, the optimal depreciation rate is lower than the efficient probability of trade, likely because the right-skew of the lognormal distribution means that the losses from excessive turnover by high value sellers outweigh the gains from eliminating inefficiently low turnover rates by low value sellers.

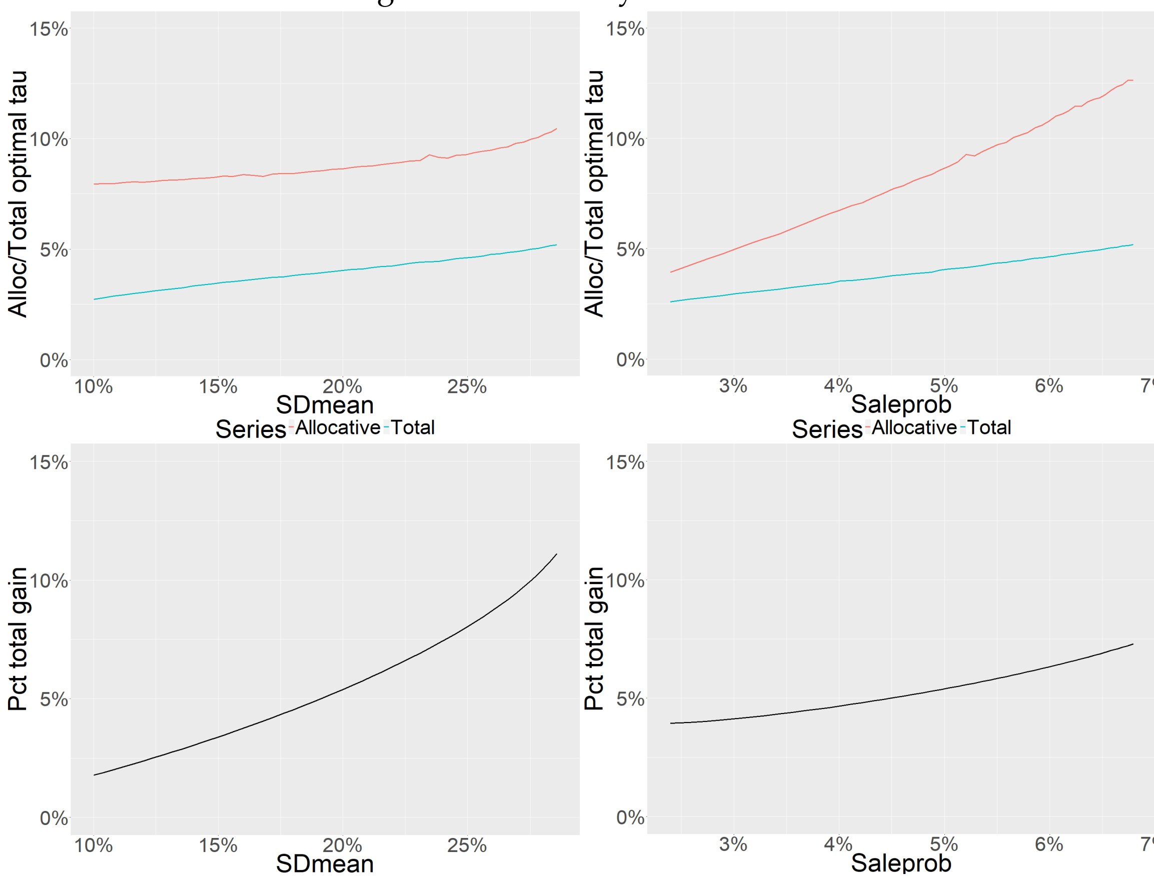
In the third panel of Figure 3 we show the behavior of various stock/flow quantities as we vary τ. License prices rapidly decrease and revenues from license fees rapidly increase as we increase τ. Intuitively, if agents have to pay license fees at rate τ every period, this is roughly equivalent to discounting by rate $\delta(1-\tau)$; thus, increasing τ has a similar effect to increasing discounting, and rapidly lowers license prices.

Figure 3: Comparative statics vs τ



Notes. In the first panel, all welfare changes are in units of percentages of the average asset transaction price at $\tau = 0$. In the third panel, license prices are average prices of licenses conditional on sale occurring.

Figure 4: Sensitivity to moments



Notes. In the left column, the $SDmean$ moment is varied holding fixed $Saleprob = 0.05$. In the right column the $Saleprob$ moment is varied holding fixed $SDmean = 0.2$. The top row shows the allocative and total optimal depreciation rates, and the bottom row shows the maximum possible total welfare gain as a percentage of the initial asset price.

Table 1: Calibration results

Invfrac	Optimal τ	Total gain	Alloc gain	Inv loss	$\tau = 2.5\%$ gain
0%	8.7%	12.9%	12.9%	0.00%	9.32%
10%	7.0%	10.6%	11.4%	-0.79%	8.21%
40%	4.0%	5.4%	6.6%	-1.24%	4.94%
70%	2.1%	1.8%	2.5%	-0.68%	1.77%

Notes. All gains are in units of percentages of the average asset transaction price at $\tau = 0$. All columns show welfare changes from the optimal τ, except for the last column, which shows the total welfare gain from imposing a 2.5% rate.

Table 2: Alternative specifications

Transition process	Optimal τ	Total gain	Alloc gain	Inv loss	$\tau = 2.5\%$ gain	% max gain
Baseline	4.0%	5.38%	6.62%	-1.242%	4.94%	83.5%
Mixbeta	3.1%	3.37%	4.17%	-0.796%	3.31%	73.9%
Jump	1.9%	1.86%	2.18%	-0.328%	1.76%	52.9%

Notes. All gains are in units of percentages of average asset prices at $\tau = 0$. For all specifications, we assume investment is 40% of average asset value. The "% Max Gain" column shows what fraction of total possible allocative welfare gains, under the social planner's optimum assuming all welfare-improving trades are made, are captured by the allocatively optimal depreciation rate.

In Figure 4, we vary the input moments used in the calibrations for the smooth transition process, and show that optimal depreciation rates and total welfare gains depend on input moments in intuitive ways. Changing the $sdmean$ moment moves the total welfare gain, with a relatively small effect on the optimal depreciation rate. Changing the $saleprob$ moment moves the optimal depreciation rate, with a smaller effect on welfare gains.

In Table 1, we show results for different choices of the fraction of investment value in asset prices. The optimal depreciation rate ranges from 2.1% to 8.7%. Total gains from depreciating licenses range from 1.8% to 12.9%. In all cases, setting the depreciation rate equal to 2.5%, or half the license turnover rate in existing markets, achieves most of the welfare gains from the optimal rate.

In Table 2, we report results from other specifications for the transition process $G(\gamma' \mid \gamma)$. Qualitatively, depreciating licenses perform worse under transition processes which induce more dispersion in seller values. However, for all transition processes we tried, depreciating license gains can achieve at least 50% of all achievable welfare gains, amounting to at least 1% of asset prices. Moreover, depreciation rates set at 2.5% achieve most of the gains from setting the optimal depreciation rate.

Based on our calibration results, a simple rule-of-thumb would be to set depreciation rates for assets in a given class at roughly half the rates of trade in markets for similar privately owned assets. From Table 1, across most specifications, the optimal depreciation rate is close to the private market trade rate of 5%, but a rate half this achieves most welfare gains. The probability of trade in equilibrium when $\tau = 0$ is lower than the socially optimal probability of trade; by further halving this amount, such a rule will tend to choose depreciation rates significantly below the allocative optimum. However, total welfare is increasing and concave in τ for depreciation rates between 0 and the optimal rate, so any depreciation rate smaller than the optimal value is welfare-improving relative to pure private ownership, and in fact fairly low rates can capture a large fraction of all possible welfare gains from depreciating licenses.

5 Extensions

In this section, we discuss several extensions that relax and examine various assumptions of our model. All these extensions build off of the two-stage model of Section 2. In Subsection 5.1, we discuss depreciating licenses in the context of multiple agents and items. In Subsection 5.2, we discuss our assumptions regarding observability of investment. In Subsection 5.3, we discuss our assumptions regarding additive separability of allocative and investment welfare. In Subsection 5.4, we show that depreciating licenses lowers, but does not distort, private-valued investments.

5.1 Multiple agents and goods

Our analysis above assumes that there is a single potential buyer of the license. In many settings, several bidders may be competing to buy the license. In such settings, we might implement depreciating licenses by allowing potential buyers to participate in an auction for the asset, with reserve price equal to the value announcement of the current asset owner. Here, we analyze optimal depreciation rates in such an auction model.

Suppose the license belongs to S, and assume for simplicity that $\gamma_S = 0$. There are multiple potential buyers $B_1 \dots B_n$, with values drawn i.i.d. from distribution F. The license is sold in a second-price auction, where S can set a reserve price p. S pays a license fee based on her reserve price p. Let y_1 represent the highest bid, and let y_2 represent the second-highest bid. S's objective function is

$$\pi_S = y_2 1_{y_1, y_2 > p} + p 1_{y_1 > p > y_2} + \eta 1_{y_1, y_2 < p} - p\tau.$$

Taking expectations over y_1, y_2 and then taking derivatives with respect to p yields

$$\frac{d\mathbb{E}\left[\pi_S\right]}{dp} = P\left(y_1 > p > y_2\right) - \tau - m\frac{dP\left(y_1, y_2 < p\right)}{dp},$$

where, as in Section 2, we define the markup $m \equiv p - \eta$.

Setting this to 0 and rearranging, we have that

$$m\frac{dP\left(y_1, y_2 < p\right)}{dp} = P\left(y_1 > p > y_2\right) - \tau.$$

This implies that the markup m depends on the difference between τ, the depreciation rate, and $P\left(y_1 > p > y_2\right)$, the probability that the asset is sold at the reserve price, rather than the total probability of sale. Intuitively, this is because S's reserve price announcement only affects her profits if the reserve price is binding. Thus, if there are multiple buyers, allocatively optimal depreciation rates can be lower than the probability of sale. In the limit as buyer competition eliminates the welfare loss from seller markups, optimal depreciation rates decrease to 0. However, many of the assets we consider trade infrequently in secondary markets, hence it is unlikely that competition between high-value buyers will be sufficient to eliminate all trade inefficiencies.

Throughout the paper thus far, we have studied trade of a single asset. We imagine this as a reduced-form model of trade for multiple differentiated products; for example, agents may consider purchasing spectrum use rights at different frequencies in different regions, or oilfields with different capacities and drilling costs. In practice, both buyers and sellers have outside options to purchase other similar assets, though the seller may incur transaction or adjustment costs for doing so. The difference between the maximum willingness-to-pay of buyers and the minimum acceptable sale price to sellers in our reduced-form model takes into account outside options and transaction costs for both parties. An important simplification of this model, however, is that we do not consider potentially complex preferences for bundles of goods. These cases are potentially important, but complex, and we thus leave these issues to future work.

5.2 Observability

Throughout the paper, we have assumed that investment is commonly observed by agents; in such a setting the government could conceivably achieve first-best investment incentives by directly rewarding observed investment. We artificially assume away this possibility.[5] We believe that this assumption is justified in the context of license design. The government structures licenses for resources such as radio spectrum to dictate the rules of asset use and trade over long time horizons; it is difficult to predict optimal uses and investment for these assets as the technological and competitive environment changes over time. Moreover, license design often simultaneously affects large classes of heterogeneous assets; the nature of optimal investment may be very different, for example, for oilfields or radio spectrum at different geographical locations. While the local costs and benefits of common-valued investments, such as oil wells

[5] We note a connection to the literature on property rights and incomplete contracts, which similarly assumes contractual incompleteness. Two papers discussing these assumptions are Maskin and Tirole (1999) and Hart and Moore (1999).

or radio antenna, are likely relatively well-observed to market participants, asset heterogeneity makes it essentially impossible for the government to observe and enforce optimal investment separately for each individual asset. Our mechanism is a simple system that gives users of heterogeneous assets robust, though suboptimal, incentives for investment.[6]

Government policy towards resource rights has attempted to directly enforce common-valued investment to the extent possible; examples include the FCC's complex buildout and coverage requirements for radio spectrum licensees (2017*a*), and various systems of quantity limits and minimum size requirements for recreational fishing (2017*b*). If common-valued investment is observable and homogeneous enough to be directly enforced, property rights are not necessary for providing investment incentives, and total welfare is maximized by selling use rights in frequent auctions. This is effectively a rental system, as is commonly used to allocate durable goods such as cars and hotel rooms.

A related issue is the interaction of depreciating licenses with asymmetric information between buyers and sellers, that is, the lemons problem (Akerlof, 1970). While this is an important and potentially complex issue, we will not analyze it in this paper. We briefly mention a simple extreme case – in settings where asset quality is imperfectly observed but persistent over time, depreciating licenses decreases the potential losses to the buyer from purchasing a lemon, since the depreciating license has a significantly lower price than a perpetual license. Hence, the total monetary risk to buyers from unobserved asset quality is lower for assets owned under depreciating licenses, which may decrease the extent of market breakdown due to the lemons problem.[7]

5.3 Additive separability

We have analyzed the additively separable model for simplicity, but our results are mostly robust to relaxing this assumption. Since the second-stage allocative game is played conditional on arbitrary first-stage investment, the net trade property holds irrespective of the structure of investment. We modelled common-valued investment as additive to both buyer and seller values, and we analyze private-valued investments in Subsection 5.4 below; these two can be combined to approximate any kind of investment whose values to agents do not depend on their types.

Considering investments with differential effects on different agent types is significantly more complex, and is outside the scope of this paper. However, we can briefly discuss a few simple settings in which allocation efficiency and investment are partially complementary. In some settings, while agents may face similar investment costs, agents who have higher values

[6]This argument loosely relates to the literature on complexity-based foundations for incomplete contracts (Segal, 1999), as well as recent work on robustness-based justifications for simple contracts (Carroll, 2015).

[7]This echoes George (1879), who argues that high taxes on land would decrease the prevalence of land speculation.

for assets may optimally make larger investments into the assets. For example, more popular wireless carriers may optimally invest more in building efficient spectrum infrastructure. In other settings, agents may have different investment cost functions, which may be the main driver of what we refer to as allocative welfare. For example, the amount of oil in a given region may be fixed, but different firms may have different extraction costs. We believe that the economic analysis of such settings is essentially the same as the model we study here. However, if allocative efficiency is linked to the optimal level of investment, improving allocation through depreciating licenses will tend to increase investment in the asset, counteracting the negative direct effect of depreciating licenses on investment. Hence, one could conceivably find that assigning assets using depreciating licenses actually increases the value of total investments made by license holders.

5.4 Selfish investments

Suppose that S can invest in increasing her private value for the asset: she can increase her own use value for the asset by λ at cost $c(\lambda)$. As before, B has value $\eta + \gamma_B$ for the asset, and all other features of the game are identical to those in Section 2. Fixing γ_S and λ, S's second stage profits are once again:

$$\pi_S(\lambda, \gamma_S, \tau) \equiv \max_q p(q)\, q + (\eta + \gamma_S + \lambda)(1-q) - p(q)\,\tau.$$

Let $q^*(\tau, \gamma_S)$ represent S's choice of q for any given τ, γ_S. In the investment stage, S chooses λ to maximize $\pi_S(\lambda, \gamma_S, \tau) - c(\lambda)$. But, using the envelope theorem, we have that

$$\begin{aligned}\frac{d\pi_S(\lambda, \gamma_S, \tau)}{d\gamma} &= \frac{\partial}{\partial\lambda}\left[p\left(q^*(\tau,\gamma_S)\right) q^*(\tau,\gamma_S) + (\eta+\gamma_S+\lambda)\left(1-q^*(\tau,\gamma_S)\right) - p\left(q^*(\tau,\gamma_S)\right)\tau\right] \\ &= 1 - q^*(\tau,\gamma_S).\end{aligned}$$

Hence, the first-order condition for S's choice of private-valued investment λ is

$$c'(\lambda) = 1 - q^*(\tau, \gamma_S).$$

This equation defines the constrained efficient level of investment, conditional on S keeping the license with probability $1 - q^*(\tau, \gamma_S)$. In words, depreciating licenses leads license owners to set lower prices and sell their licenses more often; owners correspondingly reduce private-valued investments, in a manner that is both privately and socially optimal. Hence depreciating licenses do not distort license holders' choices of private-valued investments.

In addition to private-valued investments, we might also consider investments by license owners that affect the value of the asset to potential buyers, but not to the owners themselves. A natural objection to depreciating licenses is that license owners who are unwilling to sell their assets may set low prices to minimize license fee payments, and then purposefully damage their assets, making them less attractive to buyers in order to deter purchase. However, at optimal or rule-of-thumb depreciation rates, τ is smaller than the probability of sale $q^*(\tau, \gamma_S)$ for most license owners. Thus, most license owners are net sellers of their assets in any given period; if marginal buyers' values increase, license owners gain more from increased sale prices than they lose from increased license fee payments. As a result, most license owners have net positive incentives to make investments which increase the value of the asset only for potential buyers. Another implication is that, under reasonable depreciation rates, most license owners set prices above their values, and thus receive higher total utility from selling their assets than keeping them; hence most trades induced by depreciating licenses will make both buyers and sellers better off.

Certain assets, such as internet domain names, may have relatively homogeneous low values to all potential owners ex ante. Most of the value from these items comes from private-valued investments that their owners make; for example, websites build brand capital as their users become accustomed to visiting a given domain name over time. In such contexts, both the socially efficient probability of asset trade and the turnover rates of assets in market equilibrium will tend to be low. Rule-of-thumb choices of depreciation rates will be correspondingly low. While depreciating licenses cannot greatly improve allocative efficiency in such contexts, they will not lead to serious adverse effects on the efficiency of asset trade. Depreciating licenses with rule-of-thumb depreciation rates are thus adaptive to the primitives of asset markets, playing a large role only when high equilibrium turnover rates suggest that efficient dynamic reallocation is an important concern.

Conversely, in a world in which depreciating licenses were ubiquitous, the overall level of private-valued investments that organizations and individuals make in assets that they use may decrease significantly. For example, if spectrum use rights at different frequencies were traded regularly in liquid markets for depreciating licenses, device makers would have increased incentives to develop devices that are interoperable at many frequencies, rather than tuned to any specific frequency band that they have acquired long-term use rights for. Similarly, if cars were mostly rented or held under depreciating licenses, individuals would tend to invest less in personalizing the cars that they use. Our arguments in this subsection suggest that these behaviors are efficient responses to lowered probabilities of long-term asset use. Many religious and social thinkers, especially from the Buddhist tradition, have suggested that capitalism leads individuals to develop excessive attachments to material possessions; our arguments suggest that these attachments are not fundamental to market-based systems, but are instead a consequence of private ownership and the frictions that it creates for efficient exchange. The relationship between individuals and the material assets that they use may develop quite differently in market economies augmented with alternative systems of property ownership.

6 Discussion

In this section, we relate our proposal to previous economic analysis and practices related to property rights in mechanism design, asset taxation, and intellectual property.

6.1 Mechanism design

This paper is inspired by a body of work that analyzes the role of property rights in asymmetric information bargaining problems (Cramton, Gibbons and Klemperer, 1987; Segal and Whinston, 2011). However, in contrast to the mechanism design approach adopted by these papers, we propose and study a particular license scheme. The benefit of this approach is that depreciating licenses are intuitive and simple to implement. Administering depreciating licenses involves only collecting license fees and maintaining a register of asset valuations. The only tuning parameter is the depreciation rate, which can be chosen appropriately by targeting the observable rate of license turnover, and Both the practical implementation and the tuning are much simpler than second-best procedures from the mechanism design literature.

The simplicity of our mechanism is not without its costs. Depreciating licenses do not achieve the full efficiency that is attained by the protocols described by Cramton, Gibbons and Klemperer and Segal and Whinston. In particular, one can construct settings in which depreciating licenses do not improve upon the status quo, such as the following example.[8]

Example 1. Suppose buyer values are uniformly distributed on $[0, 1]$. Sellers have value 1 with probability 0.5 and 0 with probability 0.5. In the status quo, buyers and sellers can trade at a fixed price of 0. The status quo thus induces efficient trade; one can show that any positive depreciation rate leads to inefficient trade.

Similarly to our calibration, this example shows that depreciating licenses functions poorly in settings where seller values are very heterogeneous, hence the optimal depreciation rate is very different for different seller types. In settings in which the status quo is relatively efficient, losses from imposing the extensive form depreciating licenses may outweigh any potential gains.

Thus, an important question for assessing the potential gains from depreciating licenses is the efficiency of secondary-market asset trade in the status quo. Modern policy on radio spectrum allocation has largely assumed secondary markets to be fairly inefficient. The decision to use auctions rather than lotteries to allocate spectrum was based on early thinking about the failure of the Coase theorem (Coase (1959), Cramton et al. (2011)), and the recent incentive auction for reallocating TV spectrum to new uses highlights the difficulty of reallocating resources held under long-term or perpetual rights. Thus, we believe there are at least substantial potential efficiency gains from implementing mechanisms such as depreciating licenses. Nonetheless,

[8]We thank an anonymous referee for inspiring this example.

depreciating licenses represent only a first step in the direction of practical partial property rights mechanisms, and we leave the study of other possible schemes to further research.

6.2 Asset taxation

The depreciating license system resembles a class of proposals for self-assessed taxation of land and other assets. Discussion of such self-assessment systems dates back to at least ancient Rome (Epstein, 1993). More broadly, a number of schemes have aimed to use market outcomes to assess the values of assets for tax purposes; an example is the common practice of setting property taxes based on the most recent sale price of a house.[9] A problem common to most market-based valuation schemes is that nontrivially large taxes based on market outcomes influence the behavior of market participants. In the case of self-assessed taxation, higher tax rates lower value announcements and increase license turnover rates. Viewed as an instrument for raising revenue, this appears to be an undesirable side effect. Levmore writes that "It is perhaps unfortunate that these... effects to self-assessment [on turnover rate] exist," and other critiques of self-assessed taxation (Epstein, 1993; Chang, 2012) also suggest that the effects of self-assessment on market outcomes are generally undesirable.

Our proposal inverts the classical argument for self-assessment: rather than using self-assessment to value assets for taxation, we propose a license design based on self-assessed license fees to increase the efficiency of license trade. We show that the "side effects" of self-assessment are governed by the simple economic intuition of the net trade property: asset owners announce prices above or below their values depending on whether the self-assessed tax rate is lower or higher than the probability of license turnover. Self-assessment is thus difficult to use as an tool for fully truthful value revelation for tax purposes, since no fixed tax rate gives all license owners incentives for truthful value announcement. However, self-assessment in the context of depreciating licenses can robustly improve allocative efficiency, since any license fee rate lower than the license turnover rate will induce most asset owners to announce prices closer to their values.

We are, however, not the first to study the effects of taxation on the efficiency of asset allocation. To our knowledge, Jevons (1879) and Walras (1896) were the first to suggest that common ownership could improve allocative efficiency. This idea was further explored by George (1879), who discusses land speculation and its effects on allocative efficiency. More recently, Tideman (1969) highlights how self-assessed taxes might increase turnover of property, but does not does not consider the impact of self-assessed taxation on investment or explicitly model allocative efficiency. We believe we are the first to propose assigning resource use rights using a license design that incorporates self-assessed license fees.

While we have focused on self-assessment in the context of depreciating licenses, we believe

[9] California's Proposition 13 is partially based on this principle; see for example Wasi and White (2005).

that self-assessment could also be a useful scheme for capital taxation in settings where asset valuation is difficult. Self-assessed taxation functions similarly to existing methods for capital taxation, raising revenue while decreasing investment incentives; however, it has the additional benefit of improving the efficiency of asset allocation. It may also be simpler and more transparent that existing systems, since it requires fewer of the discretionary judgments of capital values that plague the enforcement of capital taxes.

6.3 Intellectual property

Since intellectual property is non-rivalrous in consumption, the investment-allocation tradeoff from property rights is particularly clear: the socially optimal allocation is to allow all parties to use all innovations at no cost, but such a system gives innovators no incentives to develop innovations. A fairly large literature has addressed the question of optimal ownership rights over intellectual property, largely finding that partial ownership systems, such as limited-term patents, are optimal. In a sense, our argument in this paper is that a similar allocative-investment tradeoff is relevant for many assets which are rivalrous in consumption, and a similar partial ownership system, depreciating licenses, improves on the extremes of full private or common ownership.

Depreciating licenses qualitatively resemble patent buyout schemes (Kremer, 1998; Hopenhayn, Llobet and Mitchell, 2006), under which innovators' exclusivity rights can be purchased by entrants or the government under certain circumstances. However, relative to the resource rights that are our main focus, the nonrival nature of intellectual property significantly changes the economics of license design. Allocative efficiency for rival goods is approximately summarized by the turnover rate, which is bounded between 0 and 1. In contrast, allocative efficiency for nonrival goods depends on multidimensional features of downstream markets (Weyl and Tirole, 2012); market size may vary by orders of magnitude for observably similar assets. Research and development costs for innovations may be similarly difficult to predict. Hence, many of the features that lead depreciating licenses for rival goods to be fairly easy to implement and tune do not apply to intellectual property.

The formal model we work with is quite similar to Hopenhayn, Llobet and Mitchell (2006), who discuss optimal allocation of exclusive monopoly rights to sequential innovators. Hopenhayn, Llobet and Mitchell work with a somewhat simpler model than ours, as innovators' types are constant over time rather than evolving as Markov processes. This allows them to fully characterize the second-best policy in their model, in contrast to our approach of proposing the observable license turnover rate as an approximate sufficient statistic for optimal policy. However, the formal similarity between their model and ours suggests that more sophisticated schemes, such as presenting sellers with distribution-dependent menus of buyout options, may also be able to improve on the depreciating license scheme we describe here.

6.4 Alternative license designs

Use rights for most natural resources are typically assigned using term-limit licenses. The tradeoff in the choice of term limit is similar to the choice of depreciation rate in our scheme: shorter term limits improve allocative efficiency at the cost of investment incentives. However, depreciating licenses are likely to dominate term limit licenses, in the sense that they achieve better allocative efficiency for a given level of investment incentives, and vice versa.

Term limit licenses can be thought of as giving full property rights to owners for a set period of time and no property rights thereafter. Once a term limit ends, full static efficiency is attained in the auction of the following term-limit license, but the previous license holder has no stake whatsoever in the common value of the asset. This may be undesirable in many settings – for example, a fishery licensee at the end of her term has no stake in the future of the fishery, hence may dramatically overfish to the point of destroying the resource. In contrast, depreciating licenses can achieve significant allocative efficiency gains while maintaining fairly large ownership stakes. For example, a 5% annual depreciation rate, implies that the licensee has a 95% stake in the value of the asset a year in the future, a 75% stake in the value five years in the future, and so on. Thus depreciating licenses set at allocatively optimal rates induce very high investment incentives relative to term-limit licenses.

Depreciating licenses also have the benefit of conferring stationary property rights over time, which is desirable both for investment and allocative efficiency. Under term-limit licenses, trade is fully efficient when license terms end, but license holders charge high markups in all other periods. As a result, many valuable trades are lost or delayed if high-value buyers arrive before the end of a license term. Depreciating licenses induce time-constant lower markups; in each period, some trades are lost, but buyers of sufficiently high value and sellers of sufficiently low value are able to trade quickly regardless of when they meet. Similar arguments show that small time-stationary distortions to investment incentives are generally preferable to occasional large distortions.[10]

Depreciating licenses also smoothen the income that governments receive from resource rights. Instead of large periodic lump-sum payments from auctions of term-limit licenses, governments receive a smaller lump-sum from the auction of depreciating licenses and then a flow of license fee payments over time. Relative to long-term or perpetual licenses, depreciating licenses may also benefit liquidity-constrained license buyers. The capital outlay required to purchase depreciating licenses is low, as much of the total expenditure takes the form of license fee payments, and the license is fairly easy to sell by announcing lower valuations.

While we believe that depreciating licenses are generally preferable to the existing system of

[10] Our argument resembles that of Gilbert and Shapiro (1990), who show in the context of intellectual property rights that, since the social welfare loss from increased monopoly power in any period is likely to be convex, policies which grant innovators time-invariant decreased monopoly power over intellectual property are often preferable to policies granting full monopoly power for a limited period of time.

term-limit licenses, it is conceivable that other license designs can achieve qualitatively similar benefits. **?** briefly discusses one such system: licenses are sold in periodic second-price auctions, but incumbent licensees are thought of as participating in auctions while owning a share $1-\tau$ of the licenses. If they win auctions, they need only pay share τ of the auction price, and if they lose they are refunded share $1-\tau$ of the final auction price. Qualitatively, this system functions similarly to depreciating licenses, as the main difference is that license holders participate in auctions rather than announcing reserve prices in advance. Relative to depreciating licenses, this system decreases investment incentives more for a given level of allocative efficiency;[11] though it may be simpler and more familiar to participants in settings where periodic term-limit auctions are commonly used. Many other licensing schemes seem possible, and we view this as a fruitful area for further research.

6.5 Applications

In this subsection, we briefly discuss potential applications of depreciating licenses. Posner and Weyl (2017) discuss many of these applications and the relationship to existing legal institutions, and **?** discusses the application of depreciating licenses for allocating priority access rights for the 3.5GHz radio spectrum band.

As we discuss above, we propose depreciating licenses primarily as a system for assigning use rights for natural resources, such as oilfields, fisheries, and radio spectrum. Use licenses for many kinds of natural resources are currently allocated to private firms using auctions. While a large body of work has been devoted to analyzing and improving the design of these static auctions for allocating these use licenses to firms;[12] comparatively little work has analyzed the optimal design of these use licenses. The distortions created by the term-limit license design are very visible in practice; the complexity of the recent FCC incentive auction (Milgrom and Segal, 2015) derives in large part from the fact that it must simultaneously purchase licenses from existing spectrum owners and resell them to new buyers. If governments instead sold resource use rights under depreciating licenses, firms that win licenses in static auctions would set lower markups in future periods, increasing trade frequency and thus dynamic allocative efficiency without further centralized organization.

For certain kinds of nonphysical resources, such as internet domain names, there may be relatively few ways in which agents can affect the common value of the resource. For such assets, in the context of our model, full allocative efficiency can be achieved by running auctions to rent the asset in each period, thus depreciating licenses is dominated. Our argument in Subsection

[11] Intuitively, under this mechanism, full allocative efficiency requires sellers to bid their true values, which requires setting $\tau = 1$ regardless of underlying value distributions. In contrast, under depreciating licenses, in our calibration $\tau = 0.05$ is enough to give most seller types incentives for approximately truthful value announcements.

[12] Examples include timber (Baldwin, Marshall and Richard, 1997; Athey and Levin, 2001), oil and gas drilling rights (Porter, 1995), and wireless spectrum (Milgrom, 2000).

5.4 above suggests that private-valued investment incentives should not be distorted by this system of frequent rental auctions. Nonetheless, depreciating licenses might be simpler to use in practice, given the practical difficulties of frequently running centralized auctions. Moreover, concerns about common value may have some relevance even for nonphysical resources – for example, if an owner of a valuable Internet domain name invests in advertising, the persistent increase in website traffic may also benefit future owners of the domain name.

Another possible application of depreciating licenses lies within "sharing economy" platforms, such as Zipcar and Airbnb. Such platforms currently operate by renting assets out to users for short-term use; as a result, users have no incentives to maintain the common value of these assets, and platforms engage in intensive and costly monitoring to prevent agents from damaging rented assets. In such contexts, platforms could instead allocate assets by establishing marketplaces for depreciating licenses over the assets they own.

Under such a system, a user who wishes to rent an asset would be required to purchase the depreciating license for the asset from its previous user; from the user's perspective, the cost of the depreciating license can be thought of like a deposit payment. While assets are in use, users periodically pay some fraction of their value announcements as depreciation rates to the platform; these license fees can be thought of as self-assessed rental fees. While she uses the asset, the user sets a price at which it may be reclaimed from her and transferred to the next user; she would presumably set this price high while maximally exploiting the asset, and then lower it as she begins utilizing it less. Since users expect to resell their depreciating licenses, they have partial incentives to maintain the common value of the asset, so as to keep the resale value of the asset high. Thus, depreciating licenses could be used by sharing economy platforms to supplement costly monitoring in providing incentives for users to maintain the common value of shared assets.

7 Conclusion

In this paper, we argue the means of production should generally be owned neither in common nor privately, but rather through a mixed system that trades off the allocative benefits of common ownership against the investment incentives created by private ownership. We propose depreciating licenses – a system of ownership conditional on periodic self-assessed license fee payments – as a simple and robust implementation of partial private ownership. The license turnover rate serves as an observable approximate sufficient statistic for choosing the depreciation rate, and we suggest a simple rule-of-thumb for using the turnover rate in license design: depreciation rates should be set at about half the turnover rates of similar assets in private markets.

It would be interesting to study variants of depreciating licenses that could be applied to market power over variable-production goods markets, rather than just over indivisible and differentiated assets. For example, a government might regulate a monopolistic seller of divisible goods by requiring the monopolist to pay a tax equal to the market price she sets, multiplied by some fixed quantity M. Such a policy will lead the monopolist to announce lower prices; similarly to our net trade property, the monopolist will set price higher or lower than marginal cost depending on whether M is higher or lower than the monopolist's total quantity sold, which is observable by the government.

We have abstracted away from a number of important issues in this paper. We assumed that the common value of assets is fully observed by all participants, ignoring "lemons" problems from asymmetric information about common values. We abstract away from the repeated strategic interactions between agents that would arise in markets where the pool of potential owners is small. We assumed that only the current user of the asset can make investments that affect the common value of the asset; in settings such as land allocation, agents besides the asset owner may be able to take actions, such as emitting pollution which affect the value of an asset. We model repeated trade of a single asset; certain assets such as radio spectrum display high degrees of complementarity, which may cause much greater losses from market power than occur with single assets because of hold-out problems (Cournot, 1838; Mailath and Postelwaite, 1990; Kominers and Weyl, 2012). We hope that future research will analyze the performance of depreciating licenses in settings where these assumptions are not satisfied. Finally, this paper is only a first step in examining the possibilities for augmenting markets with alternative systems of property ownership, and we hope that future work continues to explore this promising area.

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Appendix

A Proofs and derivations

A.1 Two-stage model

Here, we prove our statement in Subsection 2.2 that Myerson (1981)'s regularity condition is sufficient for $\frac{\partial q^{\star}}{\partial \tau}$ to be finite for all depreciation rates below the efficient probability of sale $\tau = 1 - F(\gamma_S)$. Myerson's regularity condition states that marginal revenue is monotone. A monopolist seller with value γ_S for the asset has revenue $(M(q) - \gamma_S)\, q$. Taking a derivative yields $M'(q)\, q + (M(q) - \gamma_S)$. Taking the second derivative, we have

$$2M'(q) + M''(q)\, q < 0.$$

Now consider the monopolist's problem under a depreciation rate $\tau < 1 - F(\gamma_S)$. By Theorem 1, $q(\tau) > \tau$; hence, $0 < q(\tau) - \tau < q(\tau)$. We want to show the following quantity exists:

$$\frac{\partial q^*}{\partial \tau} = \frac{M'(q)}{2M'(q) + M''(q)(q - \tau)}.$$

So we have to show that the denominator is bounded away from 0. From our full-support assumptions on ϵ, $M'(q)$ exists and is negative for all q. If $M''(q) \leqslant 0$, we know $q - \tau > 0$, so $M''(q)(q - \tau) \leqslant 0$, and the numerator and denominator are both strictly negative; hence, their ratio is positive and nonzero and $\frac{\partial q^*}{\partial \tau}$ exists. So suppose $M''(q) > 0$. Then

$$2M'(q) + M''(q)(q - \tau) < 2M'(q) + M''(q)\, q < 0$$

Where we first use that $0 < q(\tau) - \tau < q(\tau)$, and then apply Myerson regularity. Hence, the denominator $2M'(q) + M''(q)(q - \tau)$ is strictly negative, and the ratio $\frac{\partial q^*}{\partial \tau}$ exists and is positive.

Now we turn to conditions on the cost and demand functions such that social welfare is a concave function of the depreciation rate. From the text, the marginal benefit of increasing the depreciation rate is $M(q^\star(\tau))\, \rho(q^\star(\tau))$ and the marginal cost is $\Gamma'(1 - \tau)\, \tau$. Recalling that $\rho = \frac{\partial q^\star}{\partial \tau}$, the second-order condition is

$$M'\rho^2 + \rho' M \rho + \Gamma''\tau - \Gamma'.$$

The first term is always negative ($\rho > 0 > M'$) and represents the quadratic nature of the allocative distortion discussed in the text. The final term is always negative as $\Gamma' > 0$ and represents the quadratic nature of the investment distortion. The two central terms are more ambiguous. However, Fabinger and Weyl (2016) argue ρ' is typically negative for most plausible demand forms (those with a bell-shaped distribution of willingness to pay, as we assume in most calibrations) and thus, given that $M, \rho > 0$, the second term is likely to be negative as well. The third term is ambiguous. By the inverse function theorem, given that $\Gamma = (c')^{-1}$,

$$\Gamma'' = -\frac{c'''}{(c'')^3}.$$

Assuming a convex cost function, this quantity is negative if and only if $c''' > 0$. Thus, a grossly sufficient condition (assuming ρ') for the first-order conditions to uniquely determine the optimal depreciation rate is that $c''' > 0$. However, note this term is multiplied by τ, which is typically below 10% in our calibrations. Thus c''' would have to be quite negative indeed to cause the problem to be nonconvex.

A.2 Dynamic net trade property

In this section, using Assumptions 1–3, we prove Theorem 2, the net trade property for the dynamic depreciating license game.

A.2.1 $V(\cdot)$ is strictly increasing

To begin with, we show, that any equilibrium $V(\cdot)$ must be increasing. This will allow us to consider only increasing candidate $\hat{V}(\cdot)$ functions for the remainder of the proof.

Claim 1. In any stationary equilibrium, $V(\gamma)$ is strictly increasing.

Proof. Consider a stationary equilibrium described by value function $V(\cdot)$. This defines an inverse demand function $p_{V(\cdot),F(\cdot)}(q)$. We will define the following Bellman operator for candidate value functions $\hat{V}(\cdot)$ for the seller's optimization problem:

$$\mathcal{R}\left[\hat{V}(\cdot)\right] \equiv \max_{q}(q-\tau)\,p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right]. \tag{4}$$

Note that $\mathcal{R}$ fixes the demand distribution $p_{V(\cdot),F(\cdot)}$ at the true equilibrium value function $V(\cdot)$, and only depends on $\hat{V}$ through the seller's continuation value $\delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$. As a result, $\mathcal{R}$ is a standard Bellman equation satisfying Blackwell's sufficient conditions for a contraction mapping.

Consider a candidate value function $\hat{V}(\cdot)$ which is nondecreasing in γ. Supposing $\tilde{\gamma} > \gamma$, the single-period value $(q-\tau)\,p_{V(\cdot),F(\cdot)}(q) + (1-q)\gamma$ is strictly higher under $\tilde{\gamma}$ relative to γ for all q, and the continuation value $\delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$ is weakly higher under $\tilde{\gamma}$, since $G(\gamma' \mid \tilde{\gamma}) >_{FOSD} G(\gamma' \mid \gamma)$. Hence, $\mathcal{R}\left[\hat{V}(\cdot)\right](\tilde{\gamma}) > \mathcal{R}\left[\hat{V}(\cdot)\right](\gamma)$, hence $\mathcal{R}\left[\hat{V}(\cdot)\right]$ is strictly increasing in γ. Hence $\mathcal{R}$ takes nondecreasing $\hat{V}$ functions to strictly increasing $\hat{V}$ functions; hence the true value function V, which is the unique fixed point of $\mathcal{R}$, must be strictly increasing in γ. □

A.2.2 The pseudo-Bellman operator $\mathcal{T}$

As we discuss in Subsection 3.3, stationary equilibria of the dynamic depreciating license game must satisfy two conditions. First, the sellers' value function must be satisfied for any γ:

$$V(\gamma) = \max_{q}(q-\tau)\,p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right) \mid \gamma\right]\right].$$

Second, the WTP distribution $p_{V(\cdot),F(\cdot)}$ must be consistent with the value function $V(\gamma)$, that is,

$$p_{V(\cdot),F(\cdot)}(q) = \left\{p:\ P_{V(\cdot),F(\cdot)}\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right) \mid \gamma\right] > p\right] = q\right\}.$$

We will define the following *pseudo-Bellman operator* $\mathcal{T}$:

$$\mathcal{T}\left[\hat{V}(\cdot)\right](\gamma) \equiv \max_{q}\,(q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right]. \tag{5}$$

The operator $\mathcal{T}$ is similar to the seller's Bellman operator $\mathcal{R}$ in Equation 4. The difference is that $\mathcal{R}$ fixes the inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$ at its true equilibrium value, whereas $\mathcal{T}$ calculates the inverse demand distribution $p_{\hat{V}(\cdot),F(\cdot)}(\cdot)$ assuming that buyers also act according to continuation value $\hat{V}(\cdot)$. We will likewise define the "candidate optimal sale probability function" $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)$ assuming continuation value $\hat{V}(\cdot)$, as:

$$q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) \equiv \arg\max_{q}\,(q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right].$$

In words, $\mathcal{T}\left[\hat{V}(\cdot)\right]$, $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\gamma_t)\right)$ and $p_{\hat{V}(\cdot),F(\cdot)}(q)$ describe the values and optimal behavior of buyers and sellers, assuming that the continuation value of a license owner of type γ in the next period is $\hat{V}(\gamma)$. Equilibria of the depreciating license game are fixed points of the $\mathcal{T}$ operator.

Since $\mathcal{T}$ characterizes the equilibrium of a game rather than a single-agent optimization problem, it is not necessarily a contraction mapping, and the standard contraction-based proofs of uniqueness in bounded discounted dynamic programs do not apply. However, equilibrium existence is not a problem – Assumptions 1, 2 and3, imply that $\mathcal{T}$ is a smooth function of $\hat{V}$, hence Brouwer's fixed point theorem implies that $\mathcal{T}$ must have a fixed point in the convex compact set of bounded $\hat{V}$ functions.

A.2.3 $\mathcal{T}$ Net trade property

In Claim 2, we show that, for any increasing candidate $\hat{V}$ function, the corresponding candidate optimal sale probability $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)$ respects the net trade property of Theorem 1. Since Claim 2 also applies to the true value function $V(\cdot)$ and policy function $q^*(\cdot)$, this proves Theorem 2, the dynamic net trade property.

Claim 2. ($\mathcal{T}$ net trade property) Suppose that $\hat{V}(\cdot)$ is strictly increasing. Then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)$ satisfies:

- If $\tau = 1 - F(\gamma)$, then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) = \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)\right) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right].$
- If $\tau < 1 - F(\gamma)$, then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) \geqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)\right) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right].$
- If $\tau > 1 - F(\gamma)$, then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) \leqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)\right) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right].$

Proof. We prove this by constructing an analogy to a two-stage depreciating license game. Fixing any increasing candidate value function $\hat{V}$, the optimization problem for a license owner with value γ is:

$$q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)=\arg\max_{q}\left(q-\tau\right)p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)+\left(1-q\right)\left[\gamma+\delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]\right].$$

Note that, by definition, we also have $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)=\gamma+\delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$. In words, $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$ is the WTP of buyer quantile $F\left(\gamma\right)$, which is just the use value γ plus the continuation value $\gamma+\delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$. Hence, we can write $q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)$ as:

$$q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)=\arg\max_{q}\left(q-\tau\right)p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)+\left(1-q\right)p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right).$$

Subtracting the term $\left(1-\tau\right)p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$, which does not depend on q, we get:

$$q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)=\arg\max_{q}\left(q-\tau\right)\left(p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)-p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right).$$

This can be interpreted as the optimization of a variable profit function from a two-stage depreciating license game, for a seller with use value $p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)$ for keeping the asset, faced with buyer values distributed as $p_{\hat{V}(\cdot),F(\cdot)}\left(q\right),\ q\sim U\left[0,1\right]$. Let $H_{\hat{V}(\cdot),F(\cdot)}\left(\cdot\right)$ represent the distribution of $p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)$; Theorem 1 implies that:

- If $\tau=1-H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$, we have $q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)=\tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)\right)=\gamma+\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$.
- If $\tau<1-H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$, we have $q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)\geqslant\tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)\right)\geqslant\gamma+\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$.
- If $\tau>1-H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)$, we have $q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)\leqslant\tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{J}}\left(\gamma;\hat{V}\left(\cdot\right)\right)\right)\leqslant\gamma+\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]$.

To complete the proof, we must show that $H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)=F\left(\gamma\right)$. Since $p_{\hat{V}(\cdot),F(\cdot)}\left(\cdot\right)$ is an increasing function, for a seller of value γ,

$$p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)\leqslant p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\iff q\leqslant F\left(\gamma\right),$$

hence,

$$H\left(p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right)=P\left[p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)\leqslant p_{\hat{V}(\cdot),F(\cdot)}\left(F\left(\gamma\right)\right)\right]=P\left[q\leqslant F\left(\gamma\right)\right]=F\left(\gamma\right).$$

While the notation is somewhat cumbersome, the intuition behind this sequence of equalities is straightforwards. The WTP function $p\left(\cdot\right)$ is an increasing function of the F-quantile q. Thus, for a license owner of type γ, the arriving buyer's willingness to pay $p_{\hat{V}(\cdot),F(\cdot)}\left(q\right)$ is lower than the license owner's own continuation value $p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))$ if and only if the arriving buyer has F-quantile lower than the license owner's quantile $F(\gamma)$. Thus, the probability $H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right)$ that the arriving buyer's WTP is lower than the license holder's WTP is exactly $F(\gamma)$.

□

A.3 Persistent investment

In order to accomodate investment, we need a nonstationary definition of equilibria in the depreciating license game. Let $\boldsymbol{\zeta} = (\zeta_0, \zeta_1, \zeta_2 \ldots)$ represent the path of common use values over time, and suppose that this is common knowledge. The use value for any agent A_t in any period is thus $\zeta_t + \gamma_t^{A_t}$. We will define the nonstationary value function $V_t(\gamma_t, \boldsymbol{\zeta})$ as the value of being a seller with type γ_t in period t, if the path of common use values is $\boldsymbol{\zeta}$. Analogously to above, we will define the inverse demand function in period t as:

$$p_{t,V_t(\cdot,\zeta),F(\cdot)}(q_t) = \left\{p_t : \; P_{V_{t+1}(\cdot,\zeta),F(\cdot)}\left[\gamma_t^{B_t} + \zeta_t + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V_{t+1}\left(\gamma_{t+1}^{B_t}, \boldsymbol{\zeta}\right) \mid \gamma_t\right] > p_t\right] = q_t\right\}.$$

Equilibrium then requires that, in each history,

$$V_t(\gamma_t, \boldsymbol{\zeta}) = \max_{q_t}(q_t - \tau)\, p_{V_{t+1}(\cdot,\zeta),F(\cdot)}(q_t) + (1 - q_t)\left[\gamma_t + \zeta_t + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V_{t+1}(\gamma_{t+1}, \boldsymbol{\zeta}) \mid \gamma_t\right]\right]. \quad (6)$$

We conjecture an equilibrium of this game of the following form:

$$V_t(\gamma_t, \boldsymbol{\zeta}) = V(\gamma) + \sum_{t'=0}^{\infty} \delta^{t'}(1-\tau)^{t'+1}\zeta_{t+t'}.$$

One can verify that if $V(\cdot)$ satisfies the "allocative" equilibrium Equation 5, then $V_t(\gamma_t, \zeta_t)$ satisfies Equation 6. Intuitively, as in the two-stage case, if the depreciation rate is τ, agent A_t only owns $(1-\tau)$ of the asset in period t. However, if the asset has some common value in period $t + t'$, agent A_t has to pay license fees t' times on the asset before enjoying its use value; hence she effectively only owns $(1-\tau)^{t'+1}$ of any common value of the asset in period t'.

For simplicity, we analyze the investment decision of the $t = 0$ agent; the problem is additive and identical for all agents in all periods, hence all agents make the same choice of investment in each period. Suppose investment level η_0 produces common value $\zeta_t = H_t(\eta)$ in the future. Agents' FOC for investment is:

$$c'(\eta_0) = \frac{\partial V_0(\gamma_t, \boldsymbol{\zeta}(\eta_0))}{\partial \eta_0}.$$

This implies that

$$c'(\eta_0) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H_t'(\eta_0), \tag{7}$$

proving Proposition 1.

B Calibration details

B.1 Persistent investment algebra

In our calibrations, we assume that investment decays geometrically at rate $\theta < 1$; that is, persistent investment η_0 generates period t value:

$$H_t(\eta_0) = \theta^t \eta_0.$$

Hence, following Equation 7 in Appendix A.3, the present value of a unit of investment is:

$$\sum_{t=0}^{\infty} \eta_0 \delta^t \theta^t (1-\tau)^{t+1} = \frac{\eta_0 (1-\tau)}{1-\delta\theta(1-\tau)},$$

and agents' investment FOCs are thus:

$$c'(\eta_0) = \frac{1-\tau}{1-\delta\theta(1-\tau)}.$$

We will suppose that the cost function is:

$$c(\eta) = \frac{\eta^2}{2(1-\delta\theta)g},$$

for some value of parameter g. This is a convenient functional form which leads to a simple analytical solution. Total social investment welfare for investment level η_0 is

$$Investment\ Welfare = \left(\eta_0 - \frac{\eta_0^2}{2g}\right)\left(\frac{1}{1-\delta\theta}\right). \tag{8}$$

The socially optimal level of investment is $\eta_0 = g$. The maximum possible investment NPV is thus:

$$\frac{g}{2(1-\delta\theta)}.$$

As we discuss in Subsection 4.1, we choose g such that the maximum possible net present value of investment is some target fraction $invfrac$ of the average transaction price.

Given some depreciation rate τ, constant for all time, the seller's FOC for investment is:

$$\frac{\eta}{(1-\delta\theta)\,g} = \frac{1-\tau}{1-\delta\theta\,(1-\tau)},$$

$$\implies \eta = g\frac{(1-\tau)\,(1-\delta\theta)}{1-\delta\theta\,(1-\tau)}.$$

We can plug this into Equation 8 to calculate total investment welfare for any given value of τ.

B.2 Numerical procedures

As we discuss in Appendix A.2, the equilibria of the dynamic depreciating license game are the fixed points of the pseudo-Bellman operator $\mathcal{T}$:

$$\mathcal{T}\left[\hat{V}\right] = \max_{q}\,(q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}\,(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right].$$

We numerically solve our calibrations by iterating $\mathcal{T}$ on grid-supported F distributions. We use gradient descent with our numerical equilibrium solver to find moments σ, β to match the $sdmean$ and $saleprob$ moments, as we describe in Subsection 4.1 of the text. Given a candidate value function $\hat{V}$ and decay rate β, we can evaluate the continuation value $\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]$ for any type γ, and thus also the inverse demand function $p_{\hat{V}(\cdot),F(\cdot)}\,(q)$. Thus, we can find the optimal sale probability $q^{*}_{\mathcal{T}}\left(\gamma; \hat{V}\right)$ for any $\hat{V}$, and thus calculate $\mathcal{T}\left(\hat{V}\right)$. Starting from a linearly increasing $V(\cdot)$ function, we iteratively apply $\mathcal{T}$ until convergence.

Once we have solved for $V(\gamma)$, this gives us equilibrium sale probability functions $q^{*}(\gamma, V)$ for every type γ. Together, the equilibrium $q^{*}(\gamma, V)$, the transition probability distribution $G(\gamma' \mid \gamma)$ and the distribution of entering buyer values $F(\gamma)$ define a ergodic Markov chain over values γ of the period-t owner of the asset S_t. We construct this transition probability matrix of this Markov chain, and solve for its unique stationary distribution, which we call $H_{\tau}(\gamma)$. We plot these stationary distributions for various values of τ in Figure 2. Total achievable allocative welfare is calculated as the average welfare from the stationary distribution of the Markov chain generated by assuming that all welfare-improving trades happen; that is, trade occurs whenever a buyer arrives with value higher than the seller.

Once we have solved for the equilibrium $V(\cdot)$, we can recover the equilibrium sale probability function $q^{*}(\gamma, V)$ and inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$, and we can use these, together with $H_{\tau}(\gamma)$, to recover the stationary averages of various quantities that we plot in Figure 3. Specifically, these quantities are averages of the following variables with respect to $H_{\tau}(\gamma)$:

- **Use value**: $\gamma + \eta$
- **Sale probability**: $q^{*}(\gamma,\ V)$

- **Quantile markup**: $(1 - q^*(\gamma,\ V)) - (1 - F(\gamma))$

- **Tax revenue**: $\tau p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$

For stationary average use values, if sale occurs in period t, we should use the buyer's use value, not the seller's, in calculating the stationary average. This is accommodated by multiplying the stationary distribution $H_\tau(\gamma)$ by a "buyer transition" adjustment matrix, which reflects the probability that seller type γ "transitions" through sale of the asset to any buyer type γ' with $p_{V(\cdot),F(\cdot)}(1 - F(\gamma')) > p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$.

For **asset prices**, we observe in the real world asset prices only for successful transactions; correspondingly, we would like to take an average of asset prices weighted by the probability of sale for each seller value γ. Thus, average asset prices in Figure 3, panel 2 are calculated as:

$$\frac{\int p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))\, q^*(\gamma, V)\, dH_\tau(\gamma)}{\int q^*(\gamma, V)\, dH_\tau(\gamma)}.$$

We include investment value in the asset price by multiplying investment flow value by a factor $\frac{1}{1-\delta(1-\tau)}$, and then adding the flow cost of investment. Note that, since license fees are collected regardless of sale, we do not weight offered prices by sale probabilities $q^*(\gamma, V)$ when we calculate average license fee revenues. Values labelled "NPV" are calculated by taking average flow values and multiplying by $\frac{1}{1-\delta}$.

For the sensitivity graphs in Figure 4, in order to vary the sdmean moment, for a grid of values of σ, we search for a value of β which keeps $\mathit{saleprob}$ at its initial calibration value while varying sdmean. Likewise, for the $\mathit{saleprob}$ graphs, we use a grid of β values, searching for σ values to vary $\mathit{saleprob}$ while holding sdmean constant. Note from Figure 3, panel 1 that welfare is quite flat about the allocative and total welfare maximizing depreciation rates. Thus, it is difficult to precisely pin down the values of optimal depreciation rates, and some numerical error is visible in Figure 4.

B.3 Transition distribution details

Here, we describe the choices for $G(\gamma' \mid \gamma)$ in different specifications of our calibration. In all cases, the transition process is multiplicatively separable with respect to current-period value γ_t; that is, if an agent has value γ_t in period t, her value in period $t+1$ is $\chi\gamma_t$ for some random variable χ. We thus describe transition distributions by describing the distribution of χ.

- In specification $\mathit{baseline}$, χ has a Beta distribution with shape parameters $20\omega, 20(1-\omega)$.

- In specification $\mathit{mixbeta}$, χ is a ω-weighted mixture of two beta distributions, with shape parameters $30 \times 0.98, 30 \times 0.02$ and $10 \times 0.25, 10 \times 0.75$.

- In specification jump, χ is a Bernoulli random variable with mean ω.

Intuitively, specification $baseline$ is a relatively smooth unimodal decay process, specification $betamix$ is smooth but bimodal, and specification $jump$ is as disperse as possible. The more disperse transition distributions induce more disperse stationary distributions of asset owner values. Depreciating licenses function more poorly when seller values are more disperse. We believe this is because the optimal depreciation rate depends on sellers' valued; when seller values are more disperse, no single depreciation rate is close to correct for all seller types, thus sellers on average have worse incentives for truthful value revelation.