# Inequality and Collective Action\*

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#### Abstract

In this paper we analyze the effect of inequality in the distribution of endowments of private inputs (like land, capital) on production efficiency through its effect on the voluntary provision of collective inputs (like irrigation, public research) that are complementary in production with those private inputs. The collective inputs can be both of the kind in which one producer benefits from their use by others (as in the case of research and extension) and the kind in which one's use detracts from potential use by others (as in the case of irrigation water from a pond). Markets in the private inputs are assumed to be imperfect, inhibiting their efficient allocation across individuals. In this context we work out how an increase in inequality in the distribution of private inputs affects the voluntary provision or use of the collective inputs and the total production surplus in this simple economy. In general we show that while production surplus increases with greater equality within the group of contributors (users) and the group of non-contributors (non-users) to the collective input, in some situations there is an optimal degree of inequality between the two groups.

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## 1 Introduction

While the literature on collective action in political science and economics is large, its interrelationship with economic inequality is a relatively underresearched area. Yet this is important in the management of environmental resources. For example, how does the reduction of land inequality through, say, a land reform affect agricultural productivity by changing the provision of collective goods like irrigation, or how does an increase in the inequality of ownership of different boat sizes of fishermen affect their total catch and profits in an unregulated fishery? In this paper we address such questions about the effects of private wealth inequality on collective action in the sense of the voluntary provision or use of collective goods; we do not, however, discuss the collective action problem involved in formulating or enforcing social rules for their use.

A collective action problem arises whenever externalities are present. Externalities arise whenever an individual does not internalize the full consequences his or her actions. They are quite pervasive in all walks of life. These could be both positive or negative. For example, by donating money to a particular cause I typically do not internalize the positive effect my contribution has on others who support the cause and this leads such contributions to be less than what is socially optimal. In contrast, if I operate a vehicle or some other machinery which creates pollution I do not internalize the negative effect my contribution has on others and this leads to such activities being carried out at a level that is higher than socially optimal. Whether an action is subject to externalities depends, among other things, on the nature of the action (supplying labor in the market versus voluntary work) and the institutions or rules of the game (farming can be undertaken by an individually owned farm or a cooperative, and in the latter case the actions of the members would be subject to externalities).

Although our framework is relevant to a general class of problems relating to voluntary provision or use of collective goods, for the sake of a common concrete anchor we shall often use the example of land reform in our subsequent exposition. Suppose producers use as inputs one private good (say, land) and one collective good (say, irrigation water) to produce a private good (say, rice). The private and collective inputs are complements in the production function. This collective good may be a public good (with positive externalities) like a public irrigation canal, or a common property resource (henceforth, CPR), like a community pond or forest, with negative externalities.

The assumption of decreasing returns, a standard one in most economic contexts, implies that the more scarce an input is in a given production unit, the higher is its marginal return. As a result, one would expect a more equal distribution of this input across production units to improve efficiency. If the market for this input operated well, then the forces of arbitrage would make sure it is allocated equally to maximize efficiency. However there is considerable evidence that suggests that the market for inputs such as land or capital does not operate frictionlessly and the private

endowment of an individual determines how much of that input she can use in her production unit.<sup>1</sup> There is a large literature showing that small farms are more efficient than large farms in agricultural sector of developing countries. This is typically advanced as one of the main arguments for land reform in terms of efficiency.<sup>2</sup> Some authors (e.g., Bardhan, 1984, Boyce, 1987) have gone one step further and argued that a more egalitarian agrarian structure is also more likely to solve collective action problems, especially those related to irrigation.<sup>3</sup>

But in the presence of collective action problems inequality of private endowments such as land or wealth may pull in the opposite direction. Indeed, in his pioneering work on collective action, Olson (1965) makes the following case in favor of inequality:

"In smaller groups marked by considerable degrees of inequality – that is, in groups of members of unequal "size" or extent of interest in the collective good – there is the greatest likelihood that a collective good will be provided; for the greater the interest in the collective good of any single member, the greater the likelihood that the member will get such a significant proportion of the total benefit from the collective good that he will gain from seeing that the good is provided, even if he has to pay all of the cost himself." (*The Logic of Collective Action*, p. 34).

We can interpret the "size" of a player with her endowment of the private input if it is complementary with the collective good in production.<sup>4</sup> Olson considered pure public goods only, which indeed have the property that only the largest (richest) player contributes. Our paper is concerned with the following questions. Is this property pointed out by Olson true for a more general class of collective goods that include both impure public goods and CPRs? If we look at welfare instead of the level of the provision of the collective good, is it possible that some degree of inequality may yield a higher level of joint surplus than perfect equality? Furthermore, is it possible for the allocation under some degree of inequality to Pareto-dominate the allocation under perfect equality?<sup>5</sup> If more than one player is involved in the provision of the public good, how would inequality within the class of contributors affect efficiency and how would inequality between the class of contributors affect efficiency?

<sup>&</sup>lt;sup>1</sup>Evans and Jovanovic (1989) analyzed panel data from the National Longitudinal Survey of Young Men (NLS), which surveyed a sample of 4000 men in the US between the ages of 14-24 in 1966 almost every year between 1966-81, and found that entrepreneurs are limited to a capital stock of no more than one and one-half times their wealth when starting a new venture.

<sup>&</sup>lt;sup>2</sup>See Ray (1998), Ch. 12 for a review of this literature.

<sup>&</sup>lt;sup>3</sup>Indeed, the limited evidence that is available on the effect of land tenure reform suggests that the productivity gains can be large (Banerjee, Gertler and Ghatak, 2002).

<sup>&</sup>lt;sup>4</sup>In Olson this is the share of the total benefit to the group that accrues to an individual player.

<sup>&</sup>lt;sup>5</sup>An outcome Pareto-dominates another outcome if no one is worse off and some are strictly better off under the former compared to the latter.

Since the private and collective inputs are complementary in our framework, the marginal return from contributing to the public good is increasing in the amount of the private input an agent has and which we are going to refer to as "wealth" in the rest of the paper. As a result, there will exist a threshold level of the amount of the private input such that only agents who have a level of wealth higher than this threshold will participate in providing the collective good while those with a lower level will free ride on the former group. This means that redistributions that increase the wealth of the richer players at the expense of non-contributing poorer players would achieve a greater amount of the public good, and other things being constant, this should increase joint surplus. In our framework this is how Olson's original argument shows up. However, his argument focuses only on the total amount of the public good and not on joint surplus. In particular, the gain from increasing the size of the collective input has to be measured against the cost arising from worsening the allocation of the private input in the presence of decreasing returns.

We show that the amount of contribution towards the provision (in the case of common property resources this has to be interpreted as extraction) of the collective input is a concave function of the endowment of the private input of the player for most well-known production functions (e.g., the Cobb-Douglas and the CES) and also that the equilibrium level of joint surplus (of both contributing and non-contributing players) is a concave function of the wealth distribution and hence displays inequality aversion. In addition, the total amount of the collective input is a concave function of the wealth distribution among contributing players. This means that initial asset inequality lowers the total provision of (pure and impure) public goods, and lowers the total extraction from the CPR. We provide a precise characterization of what the optimal distribution of wealth that maximizes joint surplus is in the case of imperfect convertibility between the private input and contribution to the collective input. We show that the joint surplus maximizing wealth distribution under private provision of the public good involves equalizing the wealth levels within the group of all non-contributing players at some positive level and also within the group of all contributing players. The contrast with the conclusions of both Olson and the distribution neutrality literature is quite sharp. The key assumptions leading to our result are: market imperfections that prevent the efficient allocation of the private input across production units, and some technical properties of the production function that are shared by widely used functional forms such as Cobb-Douglas and CES under decreasing returns to scale.

The above result takes the number of contributors to the collective input as given. It is difficult to characterize the optimal distribution of wealth when the number of contributors can be chosen.

<sup>&</sup>lt;sup>6</sup>Baland and Platteau (1997) provide some very interesting examples where richer agents tend to play a leading role in collective action in a decentralized setting. For example, in rural Mexico the richer members of the populaton take the initiative in mobilizing labor to manage common lands and undertake conservation measures such as erosion control.

A key question of interest is: does perfect equality among all players maximize joint surplus? We provide a limited answer to this question. It turns out that perfect equality among all players (i.e., inter-group inequality in addition to intra-group inequality) is not always optimal. If wealth was equally distributed among all players, the average wealth of contributing players is low and this could reduce the level of the collective good. In contrast concentrating all wealth in the hand of one player will maximize the average wealth of contributors, but will involve significant losses due to the assumed decreasing returns in the individual profit function with respect to wealth. The optimal distribution of wealth characterized above achieves a compromise between these two different forces.

The plan of the paper is as follows. In the next section we provide a brief review of the literature in economics that deals with similar issues. In section 3 we provide a formal analysis of a simple model and briefly discuss the implications of relaxing some of our main assumptions. Finally, in section 4 we provide some concluding observations.

## 2 A Review of the Existing Literature

The public economics literature has addressed the question of inequality among contributing players in some detail. A key finding is the surprising "distribution-neutrality" result for a particular class of collective action problems, namely the provision of pure public goods. These are public goods where individual contributions are perfect substitutes in the production of the public good and everyone gets the same benefit from the public good irrespective of the level of their contributions. Then in a Nash equilibrium the wealth distribution within the set of contributors does not matter for the amount of public goods provision. The intuition behind this result is explained very clearly by Bergstrom, Blume and Varian (1986). Suppose after the redistribution every player adjusts his contribution to the public good by exactly the same amount as his change in wealth and leaves the consumption of the private good unchanged. In that case the amount of the public good is the same as before and so the initial allocation is still available to all players. Those who have lower wealth because of the redistribution have a restricted budget set and would clearly prefer the previous allocation if it is still available. The budget set of those who have higher wealth because of redistribution expands, but not in the neighborhood of the original choice. In particular, now the extra options available to the player which are not dominated by options in the previous budget set involve a lower level of the public good compared to what she would receive if she did not contribute before, and higher levels of the private good. But she did contribute before, and so she is also better off with her previous choice.

<sup>&</sup>lt;sup>7</sup>Some of the contributions to the theoretical literature related to this result are Warr (1983), Cornes and Sandler (1984), Bergstrom, Blume and Varian (1986), Bernheim (1986) and Itaya, de Meza and Myles (1997).

Subsequent work has shown that the neutrality result depends crucially on the individual contributions being perfect substitutes in the production of the public good, the linearity of the resource constraints, the absence of corner solutions, and the "pureness" of the public good (i.e., the benefit received by a player must depend only on the total level of contributions, but not on her own contribution (see Cornes and Sandler, 1996, pp. 184-190 and p. 539; Bergstrom, Blume and Varian, 1986, Cornes and Sandler, 1994).<sup>8</sup> In this paper we consider three points of departure from the distribution-neutrality framework.

First, we adopt the framework of a generalized collective good of which pure and impure public goods with positive externality (e.g., roads, canal irrigation, law and order, public R&D, public health and sanitation) are particular cases. We also analyze collective goods with negative externality (e.g., forestry, fishery, grazing lands, surface or groundwater irrigation).

Second, another point of departure from the standard literature on voluntary provision of public goods is that we look not only at the level of provision of the collective good in question, but also the total surplus from the good, net of costs.

Third, the distribution-neutrality result assumes that the contributions towards the public good and the private input are fully convertible.<sup>10</sup> In practice, particularly in the building of rural infrastructure in developing countries, the contribution towards the public good often takes the form of labor. To fix ideas, let us think of the private input as capital. Then this assumption bypasses an important issue of economic inequality: labor is not freely convertible into capital. Typically, labor and capital are not perfect substitutes in the production technology, and because of credit market imperfections capital does not flow freely from the rich to the poor to equate marginal returns. We take this more plausible scenario as our starting point and examine the effect of distribution of wealth among members of a given community on allocative efficiency in various types of collective action problems (involving public goods as well as common property resources or CPR) in the presence of missing and imperfect capital markets. This is particularly important in less developed countries where the life and livelihood of the vast masses of the poor crucially depend on the provision of above-mentioned public goods and the local CPR (particularly when it

<sup>&</sup>lt;sup>8</sup>Baland and Ray (1999) consider whether inequality in the shares of the benefit players receive from a public good is good or bad for efficiency might depend on whether the contributions of the players are substitutes or complements in the production function of the public good.

<sup>&</sup>lt;sup>9</sup>See also the companion technical version of this paper (Bardhan, Ghatak and Karaivanov, 2002) where we provide formal proofs of all our results.

<sup>&</sup>lt;sup>10</sup>The distribution neutrality literature is couched in the framework of a consumer choosing to allocate a given level of income between her private consumption and contribution to a pure public good. We adopt the framework of a firm using a private input and a public input to produce some good. While not exactly equivalent, formally these frameworks are very similar and what we call the private input is similar to the private consumption good in the distribution neutrality literature.

is not under commonly agreed-upon regulations<sup>11</sup>), and where markets for land and credit are often highly imperfect or non-existent. In poor countries where property rights are often ill-defined and badly enforced, even usual private goods have sometimes certain public good features attached to them, and due to ongoing demographic and market changes the traditional norms and regulations on the use of CPR are often getting eroded. In such contexts inequality of the players may play a special role.

Our work is also motivated by the growing empirical literature on the relationship between inequality and collective good provision. For example, in an econometric study of 48 irrigation communities in south India Bardhan (2000) finds that the Gini coefficient for inequality of land-holding among the irrigators has in general a significant negative effect on cooperation on water allocation and field channel maintenance but there is some weak evidence for a U-shaped relationship. Similar results have been reported by Dayton-Johnson (2000) from his econometric analysis of 54 farmer-managed surface irrigation systems in central Mexico. In a different context, using survey data on group membership and data on U. S. localities, Alesina and La Ferrara (2000) find that, after controlling for many individual characteristics, participation in social activities is significantly lower in more unequal localities.

### 3 The Model

Suppose there are n > 1 players. Each player uses two inputs, k and z, to produce a final good. The input k is a purely private good, such as land, capital, or managerial inputs. We assume that there is no market for this input and so a player is restricted to choose  $k \le w$  where w is the exogenously given endowment of this input of a player. While we will focus on this interpretation, there is an alternative one which views w as capturing some characteristic of a player, such as a skill or a taste parameter. In contrast, z is a collective good in the sense that it involves some externalities, positive or negative. We assume that each player chooses some action x which can be thought of as her effort that goes into using a common property resource or contributing towards the collective good. Let  $X \equiv \sum_{i=1}^{n} x_i$  be the sum total of the actions chosen by the players where  $x_i$  denotes the action level of player i. The individual actions aggregate into the collective input in the following simple way  $z_i = bx_i + cX$ . The production function for the final good is given by

<sup>&</sup>lt;sup>11</sup>Agreeing upon such regulations is itself a collective action problem.

<sup>&</sup>lt;sup>12</sup>The assumption that the market for the private input does not exist at all, while stark, is not crucial for our results. All that is needed is that the amount a person can borrow or the amount of land she can lease in depends positively on how wealthy she is. Various models of market imperfections, such as adverse selection, moral hazard, costly state verification or imperfect enforcement of contracts will lead to this property.

 $f(w_i, z_i)$  and thus the profit (surplus) function of player i is  $\pi_i = f(w_i, z_i) - x_i$ .

Note that the input  $x_i$  appears twice in the profit function, once on its own as a private input, and once in combination with the quantities used or supplied by other agents. This implies that the private return to a player always exceeds the social return as long as b > 0. The input X can be a good (e.g., R&D, education) or a bad (e.g., any case of congestion or pollution). This formulation allows each player to receive a different amount of benefit from the collective input which depends on the action level they choose. In contrast, for pure public goods every player receives the same benefits irrespective of their level of contribution. This case, as well as many others (involving both positive and negative externalities) appear as special cases of our formulation as we will see shortly. Following the distribution neutrality literature we assume that the cost of supplying one unit of the collective input, is simply one, and that the production function, f exhibits decreasing returns to scale<sup>14</sup> with respect to the private and the collective inputs  $x_i$  and  $z_i$  (possible examples include the Cobb-Douglas production function or the CES production function).

We allow c to be positive, negative or zero. For technical reasons, when c is negative we need to assume that the absolute value of c is not too large. When c = 0 we have the case of a pure private good - there are no externalities. For b = 0 and c positive we have the case of pure public goods, i.e. the one on which most of the existing literature has focused. For b and c positive we have the case of impure public goods as defined by Cornes and Sandler (1996). For b positive and c negative we have a version of the commons problem: by increasing her action relative to those of the others an individual gains. These cases are summarized in Table 1 below:

Table 1		
Case	Description	Type of Good
c < 0  and  b > 0	negative externalities	commons (forestry, fishery)
$c = 0 \text{ and } b \ge 0$	no externalities	pure private good (supplying labor in own farm)
c > 0 and $b = 0$	positive externalities	pure public good (quality of air or water)
c > 0 and $b > 0$	positive externalities	impure public goods (roads, canal irrigation)

# 4 The Decentralized Equilibrium

Let us consider the decentralized Nash equilibrium allocation. By decentralized we mean there is no planner who coordinates the actions of the different players. Each player behaves independently.

<sup>&</sup>lt;sup>13</sup>We will refer to  $\Pi = \sum_{i=1}^{n} \pi_i$  as joint surplus or joint profits later in the paper.

<sup>&</sup>lt;sup>14</sup>In the companion paper we also study extensively the case of constant returns to scale.

<sup>&</sup>lt;sup>15</sup>In particular we need the assumption that  $b + cn \ge 0$  which implies that if a planner chooses the level of the collective input, she would choose a positive level of  $x_i$  for at least one player. This also ensures that the equilibrium is stable.

In a Nash equilibrium each player is choosing an action optimally given the action choices of others. Formally speaking, player i takes the contribution of the other players as given and solves:

$$\max_{x_i \ge 0} \pi^i = f(w_i, z_i) - x_i.$$

Let the function  $g(w_i) > 0$  denote the value of  $z_i$  that solves the first order condition of the above problem taken as equality:

$$f_2(w_i, g(w_i))(b+c) = 1$$
 (1)

Thus g(w) represents the level of the collective input  $z_i$  that player i would like to choose if her wealth were  $w_i$ . Notice that a player can affect the level of the collective input only partly through her own contribution, the rest depending on the contribution of the other players. This function merely gives the desired level of the collective input of a player as a function of her wealth level. Under some technical assumptions<sup>16</sup> we make the following observation which follows upon inspecting (1):

#### Observation 1

The desired level of the collective input for a player is increasing in her wealth level. If the production function displays constant returns to scale then the desired level of the collective input for a player is an increasing and linear function of her wealth. For most production functions displaying decreasing returns to scale (e.g., the Cobb-Douglas, the CES) the desired level of the collective input for a player is an increasing and a strictly concave function of her wealth.

This property follows directly from the complementarity between wealth and the collective input in the production function (i.e., a higher level of wealth raises the marginal return from the collective input) and diminishing returns with respect to the collective input. An increase in the wealth level raises the marginal return of the collective input relative to its marginal cost which is assumed to be constant and equal to one. To restore equilibrium at the individual level, given diminishing returns the amount of the collective input must increase. It immediately follows that the collective input for a player would be increasing in her wealth level. Suppose we take a unit of wealth from a rich person and give it to a poor person. We know that the contribution of the former to the collective input would increase and that of the latter would fall. Can we say which effect is going to be larger, i.e., what is the net effect? It turns out that if the production function displays constant returns to scale then the net effect is zero. The technical property of constant returns to scale implies that if both the wealth of a player and the amount of the collective input received by

 $<sup>^{16}</sup>$ All technical details and formal proofs of the results in this paper can be found in the companion paper (Bardhan, Ghatak and Karaivanov, 2002).

her are increased proportionally, the marginal return from contributing remains unchanged. As a result, the desired level of the collective input for a player is going to be a linear function of her wealth. If there are decreasing returns to scale then a proportional change in the wealth of a player and the amount of the collective input received by her leads to a change in the marginal return from contributing. For most well known cases of decreasing returns to scale production functions (such as the Cobb-Douglas, CES) the marginal return falls.<sup>17</sup> This means the poor person increases her contributions by an amount greater than the amount by which the rich person cuts her contribution down. In other words, the desired level of the collective input for a player is an increasing and a strictly concave function of her wealth.

Notice that the level of the collective input is strictly increasing in the contribution of a player. Given that the desired level of the collective input in increasing in the wealth level of a player (Observation 1), it follows that irrespective of whether we have positive or negative externalities, for a given level of contribution of other players, a richer player has a higher marginal profit from contributing than a poorer player. Then the following observations follow directly:

#### Observation 2.

- (i) For the case of a pure public good the amount of the collective input enjoyed by each player is the same, whether the player contributes or not. Therefore only the richest player contributes. This has the implication that even when the difference in the wealth between the richest player and second richest player is arbitrarily small, the former provides the entire amount of the public good.
- (ii) For the case of impure public goods or CPRs the amount of the collective input enjoyed by each player is different. Therefore, even if a player's marginal return from contributing is less than that of the richest player, she can contribute less and enjoy a lower level of the collective input and thereby still attain an interior optimum.
- (iii) For the case of pure private goods there are no externalities of any kind. In this case, as long as the wealth level is positive each player will choose a positive level of the action.

Let us denote by  $\hat{x}_i$  the optimal action choice of player i. Given that the richest player has a higher marginal return from contributing to the collective input than the poorer players, and since the marginal return from the collective input becomes arbitrarily large as its level goes to zero, the richest player will always contribute so long as her wealth level is positive. Given the definition of

<sup>&</sup>lt;sup>17</sup>For the technical conditions that ensure this property, see Bardhan, Ghatak and Karaivanov (2002).

 $g(w_i)$ , the optimal contribution of a contributing player can be written as:

$$\hat{x}_i = \frac{g(w_i) - cX}{h} \tag{2}$$

We characterize the decentralized equilibrium in the following two steps. First, for a given distribution of the private input we solve for the optimal contributions of each agent,  $\hat{x}_i$ , the total contribution X, and the joint surplus,  $\Pi$ . Second, we look for the distributions of  $w_i$ , which maximize the total contribution and joint surplus to be able to analyze the effects of inequality on these two variables.

### 4.1 Effect of Wealth Inequality on Total Contributions and Joint Profits

From (2):

$$X = \frac{\sum_{i=1}^{m} g(w_i)}{b + mc}$$

where m is the number of contributing agents in equilibrium. Notice that then Observation 1 implies that X is the sum of m concave functions and as such is a concave function itself. Moreover, as these functions are identical and receive the same weight, if we hold the number of contributors constant total contribution is maximized when all contributing agents have equal amounts of the private input. Therefore we have:

**Result 1:** The total contribution is strictly concave in the private input endowments and is maximized when all contributing agents have equal amounts of the private input.

Recall that our assumptions above imply that diminishing returns with respect to the collective input used by the i-th individual set in at a faster rate at a higher wealth level, and so the optimal level of the collective input is a concave function of the wealth level. Result 1 follows from this assumption, and the fact that the collective input used by the i-th individual is a linear function of the individual's own contribution and the contribution of other players.

To see this more clearly consider the two player version of the game where player 1 has wealth  $w + \varepsilon$  and player 2 has wealth  $w - \varepsilon$  where  $\varepsilon > 0$ . By changing  $\varepsilon$ , we can study the effect of greater inequality on the contributions of the two players. Naturally, the higher is  $\varepsilon$  the higher will be the contribution of player 1 and the lower will be the contribution of player 2. But if one player increases her contribution and the other player decreases hers, these will lead them to further adjust their contributions, and so on. To get a concrete answer we need to consider the reaction functions of the players which tell us how much one wants to contribute as a function of the contribution of

the other player. They are derived from the first-order conditions of the players, and are as follows:

$$x_1 = \frac{1}{b+c} \{ g(w+\varepsilon) - cx_2 \}.$$
  
$$x_2 = \frac{1}{b+c} \{ g(w-\varepsilon) - cx_1 \}.$$

Assume that both players are contributing in equilibrium and consider the effect of an increase in  $\varepsilon$  to  $\varepsilon'$ . The direct effect is to increase  $x_1$  and reduce  $x_2$ . For the case of positive externalities (c > 0) the indirect effects which work through the other player's contribution move in the same direction, while for the case of negative externalities, the indirect effects move in the opposite direction. The former case is illustrated graphically in Figure 1. Since c > 0 the reaction functions of both players are downward sloping. Suppose we start with an equilibrium at point A. An increase in inequality leads to a downward parallel shift of the reaction function of player 2 (denoted by  $R_2$  and  $R'_2$ ) and an upward parallel shift of the reaction function of player 1 (denoted by  $R_2$  and  $R'_2$ ). Notice that since the function g(.) is concave under our assumptions, the difference between  $g(w - \varepsilon)$  and  $g(w - \varepsilon')$  is bigger than that between  $g(w + \varepsilon)$  and  $g(w + \varepsilon')$ , i.e. the loss in total surplus resulting from reducing 2's contribution would be bigger than the gain from increasing 1's contribution. This implies that the new equilibrium (point B) lies to the southwest of the iso-contribution line through A (the heavy solid line in Figure 1) and hence the total contribution, X is decreasing in inequality,  $\varepsilon$ .

The effect of wealth inequality on X has implications which are quite different from those available so far in the public economics literature. Our analysis shows that greater equality among those who contribute towards the collective good will increase the value of X. Therefore a more equal wealth distribution among contributors will increase the equilibrium level of the collective input. In addition, any redistribution of wealth from non-contributors to contributors that does not affect the set of contributors will also increase X. In terms of the two-player example, this implies that as long as both players contribute, any inequality in the distribution of wealth reduces X. But with sufficient inequality if one player stops contributing then any further increases in inequality will increase X.

Let us now turn to the normative implications of changes in the distribution of wealth. Under the first-best, which can be thought of a centralized equilibrium where players choose their contributions to maximize joint surplus, the first-order condition for player i is:

$$f_2(w_i, z_i)(b + nc) \le 1.$$

For  $|\varepsilon' - \varepsilon|$  small, we can think of that difference as a multiple of the derivative of g.

<sup>&</sup>lt;sup>19</sup>In the above formula for X, holding m constant a redistribution from non-contributors to contributors will increase  $w_i$  (i = 1, 2, ..., m) with the increase being strict for some i.

The difference with the decentralized equilibrium is that now individuals look at the social marginal product of their contribution to the collective input, i.e.,  $f_2(w_i, bx_i + cX)(b+nc)$  as opposed to the private marginal product, i.e.,  $f_2(w_i, bx_i + cX)(b+c)$ . Then it follows directly that those who will contribute will contribute more (less) than in the decentralized equilibrium if c > 0 (c < 0). Also, the number of contributors will be higher (lower) than in the decentralized equilibrium if c > 0 (c < 0).

Therefore, for the case of positive externalities, the total contribution in a decentralized equilibrium is less than the efficient (i.e., joint surplus maximizing) level. Conversely, for the case c < 0, total contributions exceed the socially efficient level. From this one might want to conclude that greater inequality among contributors increases efficiency in the presence of negative externalities and reduces efficiency if there are positive externalities.<sup>20</sup> Indeed, the literature on the effect of wealth (or income) distribution on collective action problems have typically focussed on the size of total contributions. However, that is inappropriate as the correct welfare measure is joint surplus,  $\Pi$ .

In the presence of decreasing returns to scale the distribution of the private input across agents will have a direct effect on joint surplus irrespective of its effect on the size of the collective input. In particular, greater inequality will reduce efficiency by increasing the discrepancy between the marginal returns to the private input across different production units. In the case of negative externalities, these two effects of changes in the distribution of the private input work in different directions, while in the case of positive externalities, they work in the same direction. The following result characterizes the joint surplus maximizing wealth distribution for a given number of contributors (users), m.

Result 2: Suppose we have either positive externalities, or a small degree of negative externalities. For a given number of contributors the joint surplus maximizing wealth distribution under private provision of the public good involves equalizing the wealths of all non-contributing players to some positive wealth level and also those of all contributing players to some strictly higher wealth level.

This result shows that maximum joint surplus is achieved for both contributors and non-contributors, if there is no intra-group inequality. This is a direct consequence of joint profit of each group being concave in the wealth levels of the group members. The contrast with the conclusions of both Olson and the distribution neutrality literature is quite sharp. The key assumptions leading to the result are, market imperfections that prevent the efficient allocation of

<sup>&</sup>lt;sup>20</sup>Note however that a sufficiently large degree of inequality among contributors may reduce X below the first-best level in the c < 0 case.

the private input across production units, and some technical properties of the production function that are shared by widely used functional forms such as Cobb-Douglas and CES under decreasing returns to scale.

In the above result we did not talk about inter-group inequality. Formally, we took m as given while considering alternative wealth distributions. An obvious question to ask is, what is the joint-profit maximizing distribution of wealth when we can also choose the number of contributors, m. For example, does perfect equality among all players maximize joint surplus? This turns out to be a difficult question in general. Below we provide detailed analysis for the various possible cases of both positive and negative externalities. Let us first look at the case of positive externalities (c > 0). Suppose all players are contributing when wealth is equally distributed. Then from Result 2 we know that limited redistribution that does not change the number of contributors cannot improve efficiency. This immediately suggests the following:

#### Corollary to Result 2

Suppose all players contribute under perfect equality. Then if after a redistribution all players continue to contribute joint surplus cannot increase.

But suppose we redistribute wealth from one player to the other n-1 players up to the point where the former stops contributing. Recall that when the group size is m < n,  $X = \frac{mg(\hat{w})}{b+mc}$ . It is obvious that an increase in the average wealth of contributing players keeping the number of contributors fixed will increase X. It turns out that an increase in m holding the average wealth of contributors constant will always increase X. However, if we simultaneously decrease m from n to n-1 and increase the average wealth of contributors, it is not clear whether X will go up or not. If X goes down then we can unambiguously say that joint profits are lower due to this redistribution (for c > 0) since the effect of this policy on the efficiency of allocation of the private input across production units is definitely negative. However, if X goes up then there is a trade off: the increase in X benefits all players (since c > 0), including the player who is too poor to contribute now, but this has to be balanced against the greater inefficiency in the allocation of the private input.

To analyze the effect of wealth distribution on joint profits when some players do not contribute we restrict attention to the comparison between joint profits under perfect equality (i.e., when all players have the same wealth) and the wealth distribution that is obtained by a redistribution that leads to m contributing and n-m non-contributing players. From the discussion above, we know

<sup>&</sup>lt;sup>21</sup>Formally, this is because  $\frac{m}{b+mc}$  is increasing in m. The intuition is, the new entrant to the group of contributor will contribute a positive amount, which would reduce the incentive of existing contributors to contribute due to diminishing returns. However, in the new equilibrium X must go up, as otherwise the original situation could not have been an equilibrium.

that under our assumptions all players contribute under perfect equality. We focus on studying only the efficient wealth distributions, i.e. ones which achieve maximum joint surplus. Since any intragroup inequality among the contributors and non-contributors reduces joint surplus we assume that all m contributors have equal wealths and all n-m non-contributors have equal wealths after the redistribution. We are then able to show that:

**Result 3:** (a) For pure public goods perfect equality among the agents is never joint surplus maximizing. (b) For pure private goods perfect equality is always joint surplus maximizing.

We noted a special property of pure public goods in the previous section (Observation 2(i)), namely, even if the difference in the wealth between the richest player and the second richest player is arbitrarily small, the former provides the entire amount of the public good with everyone else free riding on her. This property is the key to explain why perfect equality is not joint surplus maximizing in this case. Start with a situation where all players except for one have the same wealth level, and this one player has a wealth level which is higher than that of others by an arbitrarily small amount. As a result this player is the single contributor to the public good. A small redistribution of wealth from other players to this player, keeping the average wealth of the other players constant, will have three effects on joint profits: the effect due to the worsening of the allocation of the private input, the effect of the increase in X on the payoff of the non-contributing players, and the effect of the increase in X on the payoff of the single contributing player. The result in the proposition follows from the fact that the first effect is negligible since by assumption the extent of wealth inequality is very small, the second effect is positive, and the third effect can be ignored by the optimality conditions. It should also be noted that this result goes through for both constant and decreasing returns to scale. The second part of Result 3 follows from the fact that when c = 0 a player will always choose  $x_i > 0$  however small her wealth level as the marginal product of contributing is very high. Then all players are contributors so long as they have non-zero wealth and it follows directly from Result 2 that perfect equality will maximize joint profits.

For the case of impure public goods (b, c > 0) under decreasing returns to scale we can provide only a partial characterization. We show that:

**Result 4:** Consider the case of impure public good subject to decreasing returns to scale.

(a) For a given number of players and a given level of externalities, perfect equality is always joint surplus maximizing for high values of the private marginal return from the action of a player. For low values of the private marginal return from the action, perfect equality is never joint surplus maximizing.

(b) For a given number of players and a given level of the private marginal return from the action of a player, perfect equality is always joint surplus maximizing when the level of externalities is low.

Two opposing forces are at work in this case - the "decreasing returns to scale" effect calling for equalizing the wealth of agents and the "dominant player" effect due to the positive externality calling for re-distribution towards the richest players as there is a positive effect on the payoffs of the non-contributing players. Each of the two effects can dominate the other depending on the parameter values. The direct effect of an increase in the richer player contribution on her own payoff can be ignored because of the optimality conditions.

While we cannot provide a full characterization of the case of decreasing returns, due to the existence of these two opposing forces, we can provide some illustrative examples using the Cobb-Douglas production function  $f(w, z) = w^{\alpha}z^{\beta}$  for a two player game. In Figures 2 and 3 we plot how the difference between joint surplus under perfect equality and under inequality (where the degree of inequality is chosen to maximize joint profits given than only one player contributes) varies with b and c for several alternative sets of values of the parameters  $\alpha$  and  $\beta$ . As we can see from the figures: (a) there is a unique threshold value for b such that perfect equality leads to higher surplus for b higher than this value and the opposite is true for b lower than this threshold; and (b) there is a unique threshold value for c such that perfect equality leads to higher joint surplus for c higher than the threshold and the opposite holds if c is lower.

Finally, we turn to the case of negative externalities, i.e., c < 0. We show that:

**Result 5:** When there are strong negative externalities perfect equality is never joint surplus maximizing.

Intuitively, joint surplus is the sum of individual surpluses ignoring the externality of a player's action on others, and the sum total of the externality terms. The former is concave in the wealth distribution but in the case of negative externalities, the latter is convex. For c close to zero the decreasing returns to scale effect dominates, i.e. joint profits are maximized at perfect equality but for low enough c (large in absolute value) the "cost of negative externality" term, which is convex, dominates and so greater inequality leads to higher joint profits.

#### 4.2 Extensions

It is important for our result that  $x_i$  and  $w_i$  are different types of goods and one cannot be freely converted into the other. Suppose instead that the individual can freely allocate a fixed amount of wealth between two uses, namely, as a private input and as her contribution to the collective

input. This is the formulation chosen by the literature on distribution-neutrality (e.g., Warr (1983), Bergstrom, Varian and Blume (1986), Cornes and Sandler (1996) and Itaya et al (1997)). This literature focuses on pure public goods, i.e., where  $z_i = cX$ . We show<sup>22</sup> that in the more general case of impure public goods the neutrality result does not go through except is some special cases. More specifically, our analysis shows that in this case, relaxing the assumption of perfect convertibility of the private input and the contribution to the collective input implies that the distribution neutrality result no longer holds. Greater equality among contributors always improves efficiency for impure public goods (i.e., c > 0) while for collective inputs subject to negative externalities, the effect of inequality on efficiency is ambiguous. In the latter case, we characterize conditions under which we can sign the effect of inequality on efficiency. Our results do not depend on the production functions being homothetic, but in the general case even with free convertibility, distribution neutrality can break down if the collective input is not a pure public good, as is well recognized in the literature (see for example, Bergstrom, Varian and Blume (1986) and Cornes and Sandler (1996)).

Above, we assumed that the private input and the public good are complements in the production function. In our companion paper we have also examined the implications of these two inputs being substitutes. As before, joint surplus goes up if wealth is equally distributed among non-contributors. Also, we cannot say for sure whether the optimal distribution of wealth involves perfect equality, or some inequality among the contributors (the poorest agents in this case) and the rest. For the intuition behind this result, notice that, those who contribute use the efficient amount of the input. Other players have more than the efficient level of the input in their production units. Any redistribution from the poor to the rich players does not affect the profit of the former as they exactly compensate for this by increasing their contribution. Since rich players have more than the efficient level of the input in their firms, normally a transfer of an additional unit of wealth would reduce joint profits since the marginal gain to the rich player is less than the marginal cost to the poor player. But every extra unit of wealth received by the rich player increases the input received by her firm by twice the amount because of the increase in the effort by the poor player and as a result it is not clear whether joint profits increase or decrease.

Finally, in our main exposition we studied the case where the player's own contribution and the total contribution of all players are perfect substitutes in determining the benefit from the collective input enjoyed by a player. We have also considered an alternative formulation in which they can be complements. Analytically, this case turns out to be quite hard to characterize even when we assume a specific form of the production function, namely Cobb-Douglas, and consider a two player game. We show that if we compare the allocations under perfect equality (both players have the same level of wealth) and perfect inequality (one player has all the wealth and

<sup>&</sup>lt;sup>22</sup>See Bardhan, Ghatak and Karaivanov (2002).

the other player has nothing) joint surplus is always higher under perfect equality for non-negative externalities. However, if there are substantial negative externalities then under some parameter values joint surplus will be higher under perfect inequality. The intuition for this result lies in the fact that when the negative externality problem is very severe then under perfect equality the players choose their actions related to the collective input at too high a level relative to the joint surplus maximizing solution. Perfect inequality converts the model to a one player game and hence eliminates this problem. On the other hand due to joint diminishing returns to the private input and the collective input, joint surplus is lower under perfect inequality compared to perfect equality if there were no externalities. What this result tells us is that perfect inequality is desirable only when the negative externality problem is severe and when the extent of diminishing returns is not too high.

## 5 Concluding Remarks

In this paper we analyze the effect of inequality in the distribution of endowment of private inputs that are complementary in production with collective inputs (e.g., contribution to public goods such as irrigation and extraction from common-property resources) on efficiency in a simple class of collective action problems. In an environment where transaction costs prevent the efficient allocation of private inputs across individuals, and the collective inputs are provided in a decentralized manner, we characterize the optimal second-best distribution of the private input. We show that while efficiency increases with greater equality within the group of contributors and non-contributors, in some situations there is an optimal degree of inequality between the groups.

The limitations of our model suggest several directions of potentially fruitful research. Our model is static. It is important to extend it to the case where both the wealth distribution and the efficiency of collective action are endogenous. For example, it is possible to have multiple equilibria with high (low) wealth inequality leading to low (high) incomes to the poor due to low (high) level of provision of public goods, which via low (high) mobility can sustain an unequal (equal) distribution of wealth. Also, in the dynamic case it will be interesting to analyze the effects of inequality on the sustainability of cooperation. Second, technological non-convexities and differential availability of exit options seriously affect collective action in the real world, and our model ignores them.<sup>23</sup> For example, the public good may not be generated if the total amount of contribution is below a certain threshold. This is the case for renewable resources like forests or fishery where a minimum stock is necessary for regeneration, or in the case of fencing a common pasture. Third, the empirical

<sup>&</sup>lt;sup>23</sup>The model of Dayton-Johnson and Bardhan (1999) examines the effect of inequality on resource conservation with two periods and differential exit options for the rich and the poor in the case when technology is linear. Baland and Platteau (1997) discuss the effect of non-convexities of technology in a static model.

literature suggests that even when the link between inequality and collective action is consistent with the results in our model, the mechanisms involved may be quite different in some cases. For example, transaction costs in conflict management and costs of negotiation may be higher in situations of higher inequality. Fourth, following the public economics literature, in this paper we focus mainly on the free-rider problem arising in a collective action setup. Here, the issue is the sharing of the costs of collective action. But there is another problem, often called the bargaining problem, whereby collective action breaks down because the parties involved cannot agree on the sharing of the benefits.<sup>24</sup> Inequality matters in this problem as well. For example, bargaining can break down when one party feels that the other party is being unfair in sharing the benefits (there is ample evidence for this in the experimental literature on ultimatum games). More generally, social norms of cooperation and group identification may be difficult to achieve in highly unequal environments. Putnam (1993) in his well-known study of regional disparities of social capital in Italy points out that "horizontal" social networks (i.e., those involving people of similar status and power) are more effective in generating trust and norms of reciprocity than "vertical" ones. Knack and Keefer (1997) also find that the level of social cohesion (which is an outcome of collective action) is strongly and negatively associated with economic inequality. Finally, we focus only on the voluntary provision of public goods and do not consider the possibility that the players might elect a decision maker who can tax them and choose the level of provision of the collective good. The role of inequality in such a framework is an important topic for future research.<sup>25</sup>

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<sup>&</sup>lt;sup>24</sup>See for example, Elster (1989).

<sup>&</sup>lt;sup>25</sup>Olszewski and Rosenthal (1999) address this question for pure public goods within the framework of the distribution neutrality literature.

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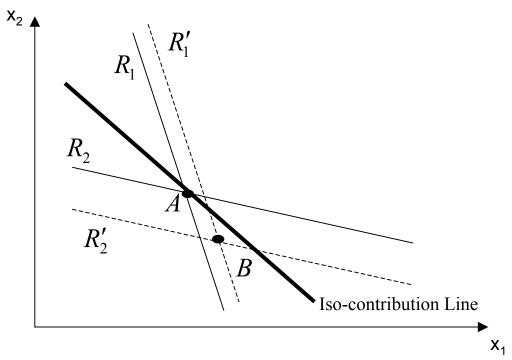


Figure 1

Figure 2: Difference in Joint Surplus Under Perfect Equality and Optimal Inequality

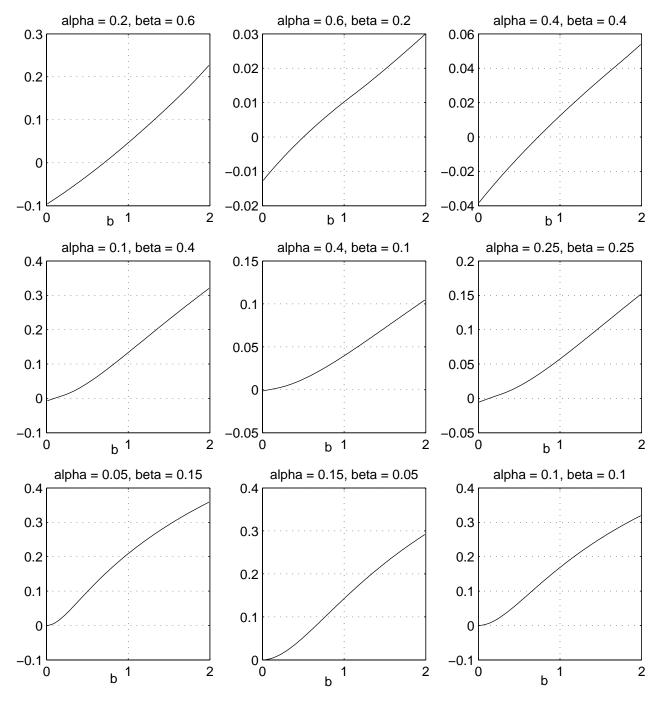


Figure 3: Difference in Joint Surplus Under Perfect Equality and Optimal Inequality

